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Fatigue of bolts in tension.

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FATIGUE OF BOLTS IN TENSION

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S. Erim

To my wife Ayla and my daughter Ege

SUMMARY

A survey has been made of the history of fatigue since the time of Wöhler and this revealed the ever-increasing number of problems caused by fatigue, ranging from complex aircraft to everyday products. Consideration of fatigue is now an essential part of design. The possibility of a fatigue failure in mechanical or structural components is usually solved by either 'ad hoc' fatigue tests or by a safety margin. Both solutions are expensive and can only be avoided by making available suitable design data based on carefully planned tests.

In this research a large number of tension bolts and screwed bar specimens were tested under constant amplitude loading over a wide range of the variables. Planning of the tests made possible an analysis which allowed statistical values to be quoted for the resulting curves. The theoretical representation of fatigue data has been studied and methods are presented which allow computer production of comprehensive fatigue design S-N curves. The required data to produce the curves is limited to the tensile strength and two constants which can be determined from a small number of fatigue tests. The principles have been demonstrated for tension bolt joints and can be extended to other types of joints.

The designer normally obtains data from 'Standards' which provide the statistically safe strength for materials and components. This research demonstrates that fatigue data can be presented to the designer in a similar manner by the introduction of two additional constants.

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Seçil Erim

Bath, England
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LIST OF SYMBOLS

A, B	Weibull's equation constants
B	Jefferson's equation constant
K	Stress intensity factor
K_c	Fracture toughness
K_f	Effective stress concentration factor
K_t	Theoretical stress concentration factor
K_{th}	Threshold for crack growth
N	Number of cycles to cause fatigue failure
N_ℓ	Logarithm of cycles to failure
$\overline{N}_\ell$	Mean of logarithms of cycles to failure
N	Material constant
P	Probability
S	Stress and Strength
S_a	Alternating stress
S_{a_o}	Fatigue limit at zero mean stress
S_e	Fatigue limit
S_m	Mean stress
S_t	Tensile strength
S_y	Yield strength
a	Crack length
σ	Coefficient of thermal expansion
γ	Confidence level

ε	Strain
ε_e	Elastic strain
ε_p	Plastic strain
$f(x)$	Distribution function of random variable x
k	One sided tolerance factor
n	Number of application of stress
m	Weibull's equation constant
r	Correlation coefficient
s	Standard deviation
S_1	Standard deviation of logarithmic values
x	Random variable
x_1	Logarithm of x
$\bar{x}_1$	Log sample mean
$\bar{\mu}_1$	Log population mean
$\bar{\sigma}_1$	Log population standard deviation
σ_f	True fracture stress
ρ	Tip radius of a crack
t_β	Specific value of variable t

CONTENTS

1. INTRODUCTION

Mechanical failures are a pervasive fact of life in our age of technology. Ranging from the failure of small items that all of us have experienced to the failure of a large complex structure that often becomes front page news they have very undesirable consequences. The large ones cause loss of life or cause serious injury to many people. The minor ones sometimes also cause loss of life or injury. Always they cause loss of valuable material and have undesirable social and economic consequences. Hence the efforts devoted to the prevention of mechanical failure are wide ranging indeed. They go from the material scientist studying fundamentals of fracture, to the metallurgist assuring that a steel forging meets specification, to the design engineer designing a new bridge. Yet despite these efforts mechanical failures continue to occur.

Definition of the problem of mechanical failure starts with its mode. Modes of failure are classified on the basis that all failures of machine and structural components are eventually traced to material failure. Following this basis, the investigation of large numbers of mechanical failures shows that the primary modes of failure are fatigue, stress corrosion, overload failures of brittle nature, overload failures of ductile nature and instability associated with buckling.

Of these modes fatigue is of primary importance. Indeed, fatigue phenomenon due to its significance in design has been of ever-increasing concern since the time of Wöhler. It is a shared belief amongst the authorities on the subject that still not less than 80% of the structural failures are fatigue failures.

In order to appreciate the problems that fatigue has brought about at every facet of engineering design up to today, a survey of historical development may be found to be quite useful. Fatigue research, when viewed from this point, has shown a changing nature. In fact the interdependence and parallelism between the varying character of fatigue study and needs of ever-growing technological design have evolved together throughout the last hundred years. At first the limited amount of research on the phenomenon of fatigue was carried out almost entirely on iron and steel, the commonly used structural and constructional materials at that time. But the difficulties and problems intensified with the advent of relatively high speed engines, turbines and machinery because the components of these new appliances were required to endure perhaps hundreds of millions of load reversals. On the other hand, the development of the aviation industry brought about its own problems. High strength/weight requirements in this field resulted in the widespread use of the alloys of aluminium.

Today various constructional and structural materials are used in design. Although engineering applications of these materials involve a wide variety of operating conditions, very seldom are mechanical components and structural elements subjected only to static loads during their entire service history. The majority of components in machinery as well as in structural applications experience variable loading conditions. Hence as soon as repeated application of stress appears on the stage so does the possibility of fatigue failure. The significance of fatigue as a design criterion lies above all in this fact. Yet despite the progress made so far fatigue failures continue to occur. This situation is due in part to the complex nature of the fatigue process and the stress material environmental interactions involved therein. However, much of the fatigue problem is simply due to poor dissemination of available information and to the lack of knowledge about 'dos and don'ts' of fatigue design. Although the likelihood of a fatigue failure should be an early consideration in the design stage it appears quite frequently as an afterthought - the result of a failure.

In making decisions about the design and operation of machines and structures proper testing to be performed beforehand constitutes a step of crucial importance. While due consideration should be given to the general principles of design against failure in a specific case under study there still seems to be no other way but to make planned experiments because of the uncertainties inherent in the material. Only so it is possible to establish the limitations of a particular material with given geometry and under specified service conditions.

Any sound generalization about fatigue performance of a material of which scatter is an inherent property should comprise:

(a) A large number of specimens tested under simulated service conditions.

(b) Analysis of experimental data by a statistical method which suits best to the nature of fatigue failure.

(c) To formulate fatigue behaviour of any given component since it is only then possible to draw significant and quantitative conclusions for design purposes.

This research is of dual character. In general the fatigue phenomenon has been surveyed in view of its historical development. In particular fatigue behaviour of rolled screwed bars and bolts in tension have been investigated.

Screwed bars and bolts are widely used in engines, in machinery of various kinds and in structures. They have unique advantages in their contribution to ease of assembly and dismantling. They also give a comparatively rigid and compact joint and from standpoint of static strength are extremely efficient.

On the other hand, the strength of these elements under dynamic loads is a matter of considerable importance, particularly in view of the increasing demand for greater output and longer life in design. Under alternating stress conditions due consideration must be given to find out the limitations and possibilities of any kind of bolt so to ensure safety and economy in design.

In this study, the bolt and screwed bar were chosen as an example of a convenient and important structural feature which could be studied in all aspects of these relevant variables. The objective of this work is to demonstrate how design data can be produced and presented such that the engineer can actually design for the fatigue requirements rather than having to correct his product when the costly failure has occurred.

2. HISTORICAL DEVELOPMENT

2.1 Early History of Fatigue

The problem of fatigue failure in engineering materials is not a recent one. It has been confronted and studied by engineers, metallurgists and scientists for far more than a hundred years. The middle of the 19th century witnessed a rapid growth of metallurgy and the invention of many new types of high speed machines. At first, the attention of engineers and metallurgists was increasingly attracted by the sudden fracture of the axles of stage-coaches and soon after by the failures of rolling stock of the rapidly developing railway systems. It was noted that such components were made of high quality ductile metal and yet the fractures were of a distinctly brittle nature with no plastic deformation. The matter appeared all the more serious in that the stress produced by the load was not only well within the ultimate strength of the material but also less than the stress which produced the first plastic deformation. Soon, however it was to be noted that these fractures of brittle appearance only occurred when the metal had been subjected to repeated alternating stresses continuing for a long time.

Although the earliest reported fatigue investigation goes back to 1829, by W. A. J. Albert (2.1-1) who did some tests on iron chains under repeated loading, Poncelet in 1839 appears to have been the first to associate the word 'fatigue' with failure under repeated loads. The first relevant paper in which this word appeared in the title was read by Braithwaite (2.1-2) before the Institution of Civil Engineers, London, in 1854. Towards the end of the first half of the 19th century structural interest in fatigue arose from the increasing use of wrought iron structures, particularly bridges, which were replacing those of brick and stone in order to meet the demands of the expanding railway systems, and in 1847 a commission was appointed in England to inquire into the suitability of iron for railway structures. Hodgkinson (2.1-3) presented his report on the matter in 1849.

However, the first of a series of papers resulting from the systematic experimental study of metal fatigue carried out by the German railway engineer August Wöhler (2.1-4) was published in 1858. His 20 year work included fatigue tests on full scale axles and on a variety of materials in smaller machines under bending, torsion and axial loading conditions. Wöhler's contribution, apart from the original ideas he brought to the mechanical design of test machines, lay in establishing reversed-stressing tests in the basic repertoire of the mechanical engineer. It may be seen quite clearly from his papers that he viewed reversed-stressing tests as an extension of the direct stressing tests and looked on the data they provide (the number of cycles to failure, N, under a reversed stress of given amplitude, S) as of the same basic significance as tensile data, such as breaking stress (as he calls it) and elongation. The relation between these two quantities, fatigue stress amplitude S and the number of cycles to failure, N, as embodied in the S-N curve, has become the basic fatigue description of engineering materials. Wohler formulated two important rules of fatigue:

a) Iron and steel may fracture under a unit stress not merely less than the static rupture stress, but also less than the elastic limit, if the stress is repeated a sufficient number of times.

b) However many times the stress cycle is repeated rupture will not take place if the range of stress between the maximum and minimum stresses is less than a certain limiting value. This limiting value has been called 'resistance in service' by Wöhler, 'natural rupture stress' by Tresca and 'natural elastic limit by Bauschinger. These three different terms were all used to denote the fatigue limit.

During the times of Wöhler's experiments, Fairbairn (2.1-5) in England carried out a series of experiments on the effect of repeated loads on riveted wrought iron girders. In 1886 Baker (2.1-6) reported the results of a series of fatigue tests on small specimens in machines similar to those used by Wöhler.

At first, following the observation that a fracture of brittle appearance occurred when a ductile material was subjected to the repeated stresses for a sufficiently long time, it was concluded that the metal degenerated and became 'fatigued' under the action of cyclical stress, rather like a human being or an animal, and that its ductile structure turned brittle. After a few decades, however, Bauschinger (2.1-7) proved that this conclusion could not be true. Bauschinger's experiments showed that there was no difference between the tensile strength and percentage elongation of the original metal and that of a sample taken from a component fractured by fatigue. So it was shown in 1890 that no degeneration or fatigue of the metal sets in as a result of repeated stressing. Contrary to animal fatigue, metal fatigue (as it is widely used today) from the engineering viewpoint, is characteristically a cataclysmic process. Detection of development before the latter stages is difficult, nor is the condition, in general, dissipated by recovery and therefore the damage is cumulative resulting in a marked distinction from animal fatigue. Bauschinger was also the first to show that the tensile yield stress is decreased as a result of previous compressive loading and that under alternating loading the loop varies with repeated loading and unloading.

By the turn of the 19th century, some eighty papers on the subject of fatigue had been published and fatigue failures had been reported in railway rolling stock axles, crankshafts, chains, wire ropes, marine propeller shafts and iron structures.

2.2 Fatigue Research 1900 to 1920

Between 1900 and 1920 much of the emphasis on fatigue research was in gathering empirical data. Ewing (2.2-1), Rosenhain (2.2-2) and Humfrey (2.2-3) studied the mechanism of fatigue with the aid of the metallurgical microscope and showed the formation of slip-bands and fatigue cracks in iron crystals which had been subjected to repeated stresses. In 1900, Gilchrist (2.2-4) put forward the hypothesis that a fatigue crack began as a result of localized stresses which exceeded the breaking strength of the metal. Bairstow (2.2-5) demonstrated the existance of hysteresis of elastic deformations and showed its connection with fatigue in 1910. In 1911 a very extensive investigation was reported to the Institution of Mechanical Engineers in London by Eden, Rose and Cunningham (2.2-6) who had investigated the effects of heat treatment, surface finish, form of test specimen and many other variables on the fatigue behaviour of both steels and non-ferrous alloys. Frémont (2.2-7) in his paper presented to the Académie des Sciences in 1919 concluded that a machine part will withstand alternating stresses indefinitely under the elastic limit and if this limit is exceeded it is the accumulation of absorbed energy which ultimately produces permanent deformation. On the basis of this theory it has been possible to improve the fatigue life of certain components by reducing their sizes at some carefully chosen points in such a way as to increase the flexibility and thus permitting them to absorb a greater quantity of dynamic work.

2.3 1921 to 1939

The period 1921 to 1939 saw the development of two main approaches to the problem of fatigue. The first approach selected by engineers was an attempt to assess the performance of components in service by relatively simple laboratory tests. The second by physicists and metallurgists, who were concerned principally with the fundamental aspects of the problem and who attempted to explain the phenomenon by changes in the internal structure of the material. Broadly speaking, the engineering approach was followed in the United States of America, Germany, France and Japan, while in England the emphasis was on the more basic aspects of the subject. A brief review of the period may contain the following main points:

The influence of the mean stress in alternating and fluctuating cycles had been widely studied and much useful data obtained. But no general relation emerged between mean stress and a safe range of stress.

In the belief that design was based entirely on a definite fatigue limit or a lengthy endurance limit little regard was paid to the shape of an S-N curve, although these were usually carefully determined by extensive tests. Also practically no attention was paid to the study of variable loadings involving stresses exceeding the fatigue limit.

The influence of test frequency was studied. At room temperature no unfavourable frequency effect was found up to the order of 10,000 cycles per minute. (2.3-1)

Concerning damping properties of the material, it had been suggested that high damping capacity was related, in some way, to fatigue strength and particularly to low notch sensitivity, but no such relations were established. This subject does not appear to be of importance in later work.

In order to reduce the long process of endurance testing several different 'short-time' methods of determining the fatigue limit had been proposed and examined. It was accepted that none of these was reliable.

As experience had shown that service failures usually start at a discontinuity of geometric form or surface weakness, extensive investigations had been made in many laboratories based on the designed or accidental cases. While there was much evidence that the full theoretical effect of a stress concentration was often realized in a static test in which the criterion was failure of simple elasticity, the result was different and complicated when the fatigue limit was concerned. Generally it was found that the effective stress concentration factor, K_f, under cyclic loading was less than the theoretical factor, K_t. But the difference varied widely from material to material also with the size and shape of the discontinuity. Further, the notch sensitivity under fatigue of a material was, in general, less pronounced for the higher values of K_t. The discovery of the presence of adverse specimen-size effects when discontinuities were present led to considerable concern. Various suggestions were put forward to account for this scale effect, but as no critical experiments were made, no conclusion could be reached.

The direct outcome of studies of stress concentration was the adaption of improved design methods aimed at reducing the effects of unavoidable stress raisers. Such methods were based largely on the hydrodynamical analogy of stress flow lines and included features namely relieving grooves and collars, improved transition fillets with or without undercutting, and the introduction of suitable and additional stress raisers to reduce the existing effect. These measures resulted a marked improvement in fatigue strength. The introduction of compressive surface stresses by means of new surface treatment methods such as surface rolling and shot-peening led to striking improvements. (2.3-2)

For the first time a start was made in the field of combined fatigue stresses. Bending and torsional stresses in phase with one another were studied. (2.3-3)

The influence of temperature on fatigue strength had received much attention. In regard to subnormal temperatures, it was established that, down to -40 deg. C., metals exhibited increasing fatigue resistance, both in the presence and absence of stress concentrations. The general conclusion was reached for the design purposes that where both fatigue failure and undue deformation must be avoided, there is, for a given material, a temperature up to which fatigue strength is of importance and above which the static 'creep' properties become more important. (2.3-4)

Due to the considerable increase in certain types of service failure where an operating part was subjected to repeated stresses in a corrosive environment, an extensive investigation had been made into the problem of corrosion-fatigue. Once the nature of the problem was recognized, rapid progress was made, mainly by the substitutions of more suitable materials and by the use of inhibitors, surface treatments and coatings on the material.

A useful start had also been made into the investigation of the trouble known as 'frettage-corrosion' or 'fretting' usually encountered at surfaces which are in intimate contact and subjected to vibration. It was established that relative surface movement was an essential condition and that local metallic seizure, with some plastic deformation,was a primary factor in the action.

The effective stress concentration of fatigue cracks was determined experimentally. Batches of specimens were subjected to unsafe ranges of stress until cracks were clearly visible; the fatigue limits of these pre-cracked specimens were then found by endurance tests. The cracks did not spread unless the nominal applied stress range exceeded 50 percent of the intrinsic fatigue limit of the material. Thus contrary to the expectations based on elastic theory, the effective stress concentration of a long, sharp crack had a maximum value of 2 only. On the practical side, the investigation resulted in a recommendation that, after a certain mileage, every railway axle was to be removed and critically examined for cracks.

At this time some full-scale fatigue tests including those on bridge assemblies and structural members, welded pressure vessels, large welded and riveted joints, oil-well drill pipes, aircraft propellers, large pipes and bends, complete rear-axle assemblies of motor vehicles were carried out.

A specialized group directed the research towards a better understanding of the mechanism of fatigue in relation to the crystalline structure. Observations of surface characteristics by employing the metallurgical microscope and X-ray precision methods established slip primarily as a hardening process for all stress ranges, safe or unsafe. Also, essentially as a local event, confined not only to certain crystals or to regions of any particular crystal, but to portions only of a crystal plane, over the whole of which a simple and complete shearing action did not usually take place. But apart from the general hardening, there was the more important secondary effect whereby, under an unsafe range of stress, deformation became intensified in areas of massed slip in which the fatigue crack formed.

By the use of X-ray precision methods it was quantitatively established that the distortion produced on crystal planes by slip was much greater in the slip direction than in the transverse direction. Under safe and unsafe ranges alike the structure broke up into a disoriented mosaic, with the production of crystallites of completely random orientation and a limiting size of 10^{-4} or 10^{-5} cm. But these effects occurred also under static straining and were accepted as the consequences of plastic deformation and the cause of strain-hardening; they were not discriminative in regard to fatigue failure. In a static test to failure, relatively large volumes of metal arrive simultaneously at the fracture condition; under fatigue that plastic deformation is reached progressively and extremely locally. A much deeper insight was thus obtained into the essential differences between deformation and fracture conditions in general and between static and fatigue failure in particular. (2.3-5)

2.4 The 1939-1945 War Years

During the 1939-1945 war years, research effort had naturally been concentrated on new problems of wartime developments and difficulties attending exceptionally severe user conditions (2.4-1)

The practice of shot-peening to improve fatigue strength or life had been considerably extended during the war (2.4-2). The need for accurate information regarding the varying loadings in service often in regions inaccessible during operation, had led to development of strain-gauges.

In this period, valuable and detailed information were collected to explain fatigue failures in service (2.4-3). Apart from manufacturing defects, the primary causes were stress concentration effects and surface defects, accelerated in some cases by corrosion and fretting; the press-fit had presented particular problems in this respect. Experience in the heavy machine industry had traced the failures to the same primary causes (2.4-4).

Much study had been given to the development and critical use of non-destructive methods of crack detection (2.4-5). The practical recommendation made was that, as there was in existence no method for determining when fatigue was in operation, a part should be immediately replaced or repaired when a crack was detected in service.

By alterations in design and the development of improved bearing alloys special problems encountered in service were solved (2.4-6) Breakages of aerial telephone lines had been traced to fatigue due to vibration resulting from wind eddies: good results had been obtained from improved joints, ties and terminations having specified damping characteristics (2.4-7). A proposed basis of design for combined stresses was put forward (2.4-8) the fatigue failures of bolts, studs and nuts with improved form of design were studied (2.4-9). The general subject of stress concentration was given much attention and various methods were developed (2.4-10). But there remained the puzzling anomalies of the variation of notch-sensitivity values, and of disturbing size effect (2.4-11).

The invention of counting accelerometers led to a very important series of measurements of the loads arising on aircraft structures during military operations (2.4-12). The loading statistics gathered from hundreds of aeroplanes of different types during thousands of flying hours gave a fair picture of the nature of varying loads arising on aircraft structures. It was then possible to say that loads of the order of three-quarters of the ultimate strength of an aeroplane wing arise about once in the aeroplane's life, whereas loads equal to one-half of the ultimate strength occur roughly 500 times. From the experimental statistical background it was pointed out that the practice of considering the strength of a structure in terms of static load or repeated loads of constant amplitude might be misleading, especially if these were based on properties of materials determined by laboratory tests on small specimens rather than on the structures themselves or on structural components. New types of investigation were required into random loading effects, including different sequences of the same loading. Joints and other regions of stress concentration were most likely sources of initiation of failure, while, in a redundant structure, partial failure of one part might transfer to another an unduly high repeated loading which it could not survive. Attention was drawn to the necessity for fatigue tests on complete structures. Finally the suggestion was made that, taking rigid maintenance, inspection, repair and ordinary replacement for granted, a considered and definite life should be assigned to an aircraft structure.

2.5 Post-war fatigue research between 1945-1960

After the war, active interest was maintained on some subjects of fatigue study, amongst which were internal stresses, selection of material by the designer with special reference to light alloys, transition temperatures, influence of metallurgical structure, and techniques of physical metallurgy in the study of fatigue.

Much experimental data were collected and evaluated in relation to theoretical and effective stress-concentration factors and to notch sensitivity (2.5-1). A three-dimensional model was proposed for fatigue under direct stress in terms of the tensile ultimate strength, which exhibited relationships between stress range, mean stress of cycle, and cycles to failure, for both plain and notch specimens.

The problem of fatigue in aircraft structures is intensified by the continuing trend of development in engine power, aircraft size, service requirements. Increased flight speeds meant in a given flying time, an increase in the number of gusts encountered and higher gust load for a given gust velocity. Developments in materials had produced higher static strength but often without proportionate increases in fatigue strengths. The serious difficulties placed on the structural designer in this exceptionally difficult branch of engineering were investigated. And in the U.S.A. a new objective of designing for a limited life, with only a small probability of failure in selected and non-vital components, while still maintaining a satisfactory strength/weight radio was developed (2.5-2). This involved the determination of repeated service loads and their statistical treatment together with a knowledge of the resistance of the structure to these loads, taking into account the response of the structure.

Due to the characteristic scatter of results, there is no exact answer to the question as to the number of cycles, N, which a specimen or a component or an assembly will endure before failure. The probability of failure P has to be introduced and this involves extrapolation, with the derivation of distribution functions of S or of N. This subject was fully studied by Weibull (2.5-3) one of the earliest European investigators in this field. A series of papers by Freudenthal and his associates on the application of statistical methods to fatigue was published in 1946 (2.5-4).

The general problem of cumulative damage which is to assess the damage suffered from a cyclic history of various stress amplitudes and, if possible, to express this in terms of the remaining capacity to resist further fatigue action was studied (2.5-5). The absence of basic knowledge on which to form a quantitative estimate of total physical damage, even in the simplest case of a straight run to fracture under a constant stress range has presented a formidable difficulty. Also there was no justification for any assumption that the damage/cycle ratio rates were similar for different values of stress range, or that the total result was independent of the sequence in which the stresses were applied.

Miner (2.5-6) put forward the hypothesis which assumed a linear rate of damage and a limiting value of unity at fracture for the arithmetic sum of cycle ratio $\Sigma \frac{n}{N} = 1$. This approach, however approximate is helpful in estimating probable life until the facts are established. Comparison with experimental data showed that with approaches to unity value, some wide variations of under and over estimation were found.

In September 1956 an International Conference on Fatigue of Metals was held at the Institution of Mechanical Engineers in London, and it provides a good starting point for reviewing the progress in research during the mid 50s. In this conference many papers of great importance were presented. For the sake of presenting briefly the state of knowledge attained for the period specified these papers were divided into the following main subjects:

Stress Distribution

It was shown, by Forrest (2.5-7) using results presented at the above-mentioned Fatigue Conference, that if a mean tensile stress is superimposed, so that the stress fluctuates wholly in tension or between unequal values of tension and compression, then the alternating stress amplitude that a material can withstand for a given endurance is reduced. The addition of a mean compressive stress, on the other hand, increases the fatigue strength. It is plausible that a tensile stress accelerates the crack propagation while a compressive stress inhibits it. Appreciation of the fact that any residual stress in the metal has a considerable influence on the fatigue strength is very important. As fatigue failures generally start to propagate from the surface the resistance of a part to fatigue failure may be reduced if there is a residual tensile stress at the surface, and may be increased by a residual compressive stress at the surface. Some improvements in fatigue strengths were obtained by residual compressive stresses introduced by a number of surface treatments such as shot-peening, surface rolling, case hardening, etc. (2.5-8). In low strength materials the beneficial effect arises mainly from the intrinsic strengthening of the surface layers, for the residual stresses tend to disappear as a result of yielding during subsequent stressing, but in high strength metals the effect of residual stress may be predominant. Matson and Roberts (2.5-9) by means of strain-peening experiments on a spring steel, showed that the great improvements could be achieved by proper residual compressive stressing. The adverse effects of some plating processes - particularly nickel and chromium - on fatigue strength were attributed to tensile residual stresses at the surface and improvements were obtained when these stresses were reduced

It might be expected, from the influence of mean stress, that fatigue failure is dependent on the maximum value of the tensile stress, but for ductile metals, it depends predominantly on the shear stress. This is evident from a comparison of fatigue strengths in bending and torsion; in addition, metallographic observations show that fatigue cracks originate within slip bends, which are produced in metal by a shearing action. Tests on thick cylinders subjected to repeated internal pressure, by Morrison and Perry, showed that the fatigue strength was dependent almost entirely on the range of maximum shear stress and not significantly on the magnitude of the triaxial tension. (2.5-10)

Design Approaches

Since the parts or structures themselves are usually subjected to loads of varying amplitude in service there are two basic approaches to design. Whenever it is possible to ensure that the fluctuating stresses never exceed the fatigue limit or long-life fatigue strength, design may be based on these respectively. For some applications, however, this is not possible and stresses sometimes greater than the fatigue limit must be expected. In such cases, fatigue failure is almost overwhelmingly likely if the part is used for a sufficiently long time therefore design must be based on a finite life. For this purpose, it would be useful to be able to predict the fatigue life under stresses of varying amplitude from the results of conventional fatigue tests at constant stress amplitude. Miner's Linear Damage Low provides the simplest basis for doing this. The assumption is made that the application of n_1 cycles at a stress range S_1 for which the number of cycles to failure is N_1 causes an amount of 'fatigue damage' $\frac{n_1}{N_1}$ and that failure will occur when $\Sigma \frac{n_1}{N_1} = 1$.

Although Miner's Law is used today and still found useful for making estimates of the fatigue life, it does not provide a sound way of determining the life under all varying stress conditions for a number of reasons: firstly, the fatigue strength of some metals, notably steel, can be considerably increased by repeated stressing below the fatigue limit; secondly the occurrence of a few heavy overloads on a component are likely to introduce residual stresses at stress raisers and; thirdly, fatigue cracks produced by high stresses can be propagated by stresses well below the original fatigue limit. Some attempts were made to derive non-linear laws by Liu and Corten (2.5-11) but in view of the above-mentioned factors, no law of general application can be expected. Instead, full-scale parts have been subjected to programme fatigue tests, to find the variations from the cumulative damage given by Miner's rule.

Crack Propagation

There had been little reliable data on the rate of propagation of cracks until 1956. An investigation at the National Engineering Laboratory showed that the early stages of crack propagation for a wide range of materials can be expressed by one basic equation The tests were made on 10 inch wide sheets, about 0.1 inch thick, containing a small central slit. These sheets were subjected to fluctuating tensile loads. It has been found that the crack growth may be discontinuous, particularly at low ranges of stress, but when the growth is continuous, the growth rate for crack lengths up to one-eighth of the sheet width can be represented by the following equation for all the materials tested:

$$\frac{dl}{dN} = \frac{lS^3}{N_S} \qquad (1)$$

where l is half the length of the crack including the initial slit, S is the nominal alternating stress on the cross-section, N is the number of cycles and N_S is a material constant which may depend on the mean stress.

The susceptibility to crack propagation did not appear to be related to other mechanical properties. As a crack proceeds a stage is reached at which the relatively slow rate of propagation described by the above equation suddenly changes to a much higher rate which quickly leads to fracture.

Frost has also made a study of the conditions governing the non-propagating of fatigue cracks (2.5-12). Cracks are sometimes found to develop at the root of very sharp notches, reach a certain size, but then not propagate further. This behaviour can be illustrated by Frost's results on notched specimens of mild steel shown in the following figure:

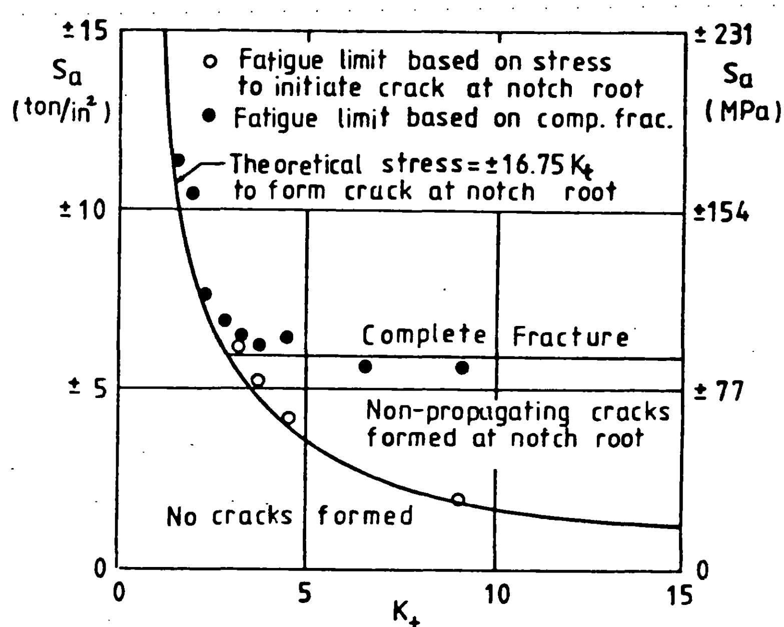

FIG. 2.1 Alternating stress against K_t for mild steel rot-bending notched specimen [Ref.(2.5-7)]

All the specimens contained notches 0.05 inch deep, but radii at the root of the notch, varying from 0.09 inch to as little as possible, were used in order to vary severity of stress concentration. A measure of this severity is given by the stress concentration factor, K_t, which is defined as the ratio of maximum local stress in the region of a discontinuity or notch, to the nominal stress evaluated by simple theory. Frost found that the nominal stress required to start a fatigue crack at the root of a notch was in good agreement with the theoretical value given by the unnotched fatigue limit, which was 16.75 t/in^2 ($\simeq$ 259 N/mm^2), divided by K_t and this is represented by the curve in Figure 1. However if the nominal stress (based on the net area) was less than 5.75 t/in^2 ($\approx$ 89 N/mm^2), the fatigue cracks travelled only a short distance from the root of the notch and then stopped. Thus however sharp the notch, these specimens showed a minimum fatigue limit of 5.75 t/in^2.

Frost carried out tests on specimens containing notches or cracks of different depths; he showed that, provided the notch or crack was small compared with the specimen size, the minimum fatigue limit (that is, the stress required to propagate the crack) was governed by the equation:

$$S^3 l = \text{constant} \qquad (2)$$

where S is the nominal alternating stress and l is the length of the crack, including the notch. In practice the length of a non-propagating crack at the root of a sharp notch is usually small, compared with the depth of the notch, so that the minimum fatigue limit can be determined approximately by substituting the depth of the notch for l in equation (2). The occurrence of non-propagating cracks was known many years before their presence was recognized in notched fatigue specimens, tested at constant stress amplitude. Such cracks are often found, for example, in locomotive axles at the press-fit with the wheel; they are a result of stress concentration or fretting corrosion. The presence of a fatigue crack in a component in service is almost always a source of danger, because the occurrence of an unusually high load may be sufficient to propagate an existing fatigue crack or even to cause complete failure by static fracture.

Low Cycle Fatigue

There are a number of applications where engineering parts are subjected to only some hundreds or thousands of stress reversals during their working life; for example, pressure vessels, the landing gear of aircraft, the working parts of guns, aero-engines. For such low endurances, ductile metals are able to withstand an appreciable repeated plastic deformation. Consequently, the stress is not directly proportional to the strain and it is necessary to draw a distinction between resistance to alternating stress and resistance to alternating strain. In practice, it is often the resistance to strain that is more important because fatigue failures almost always propagate from regions of stress concentration where the material is confined to a given strain range, by the surrounding material deforming elastically.

The range of strain, ε, that a material can withstand for a given endurance will be partly elastic and partly plastic, so that one may write:

$$\varepsilon = \varepsilon_e + \varepsilon_p \qquad (3)$$

where ε_e and ε_p are the ranges of elastic and plastic strains respectively.

In an interesting series of experiments, Coffin (2.5-7) showed that ε_p is approximately related to the endurance cycles, N, by the relation:

$$\varepsilon_p \sqrt{N} = \text{constant} \qquad (4)$$

Experimental results for a wide range of ductile metals obey this relation quite closely for endurances up to about 10^5 cycles. Moreover, the value of ε_p for failure in $\frac{1}{4}$ cycle, as computed from equation (4), agrees fairly closely with the true strain of fracture in a static tensile test. Thus, the repeated plastic strain, ε_p, a material can withstand is quite closely related to its ductility. The range of elastic strain, ε_e, it can withstand, on the other hand, is directly proportional to its fatigue strength and for short endurances this is quite closely related to its tensile strength. It is therefore to be expected that, for very short endurances when plastic strain predominates, resistance to total alternating strain will depend primarily on ductility; and it will depend primarily on strength for longer endurances, when elastic strain predominates. In fact, for quite a wide range of endurance, the strain range is almost independent of the material. This was demonstrated by Low at the Fatigue Conference (2.5-13) for a number of steels and aluminium alloys, copper, brass and phosphor-bronze and subsequently by Kooistra (2.5-14) for steels ranging from 25 t/in^2 ($\approx$ 386 N/mm^2) tensile strength. The practical implication of this result is that, for short endurance applications, little may be gained by the use of high strength materials.

High Temperature

Failures caused by repeated strains due to rapid changes in temperature appeared to be a serious problem. The phenomenon called thermal fatigue has been met where high rates of change of temperature can occur, for example, the blades and flame tubes of gas turbines. Fluctuations in temperature during the operation will produce thermal stresses, but experience has shown that failure is more likely to result from the much larger expansions and contractions when equipment is started or stopped. This was clearly demonstrated by full scale tests on a gas turbine.

The resistance of a metal to thermal fatigue was found to be dependent basically on three properties; its thermal conductivity k, its coefficient of thermal expansion α and its resistance to alternating strain ε. A high value of k is beneficial because it reduces the temperature gradients, while a low value of α is desirable because the thermal strains induced are directly proportional to it. Values of α and k are known for most metals, so that a comparison of the thermal fatigue resistance of different metals can be made if ε is determined. One method for obtaining such data was developed by Coffin (2.5-15) and has been adapted by other investigators (2.5-16). It was a thin tubular specimen which was rigidly fixed at its ends and alternately heated by a high current and air-cooled. The fluctuating temperature produced a fluctuating axial strain in the specimen and the test was continued until the specimen fractured.

As thermal fatigue failures generally result from the starting and stopping of plant, failure is usually the result of a relatively small number of stress reversals. Some evidence has been found that when the temperature is fluctuating, the value of ε is about the same as if the temperature were held at the maximum all the time. In other words, the resistance to thermal fatigue may be thought to be directly related to resistance to alternating strain at the maximum temperature.

Fretting Corrosion

There was an increased interest in fretting corrosion in order to understand its action. It was found that in the early stages of fretting, the damage is similar to the wear resulting from unidirectional sliding. As fretting proceeds, more wear particles are produced and, owing to the small relative movement, these cannot escape but build up between two surfaces (2.5-17). Metal to metal contact ceases at an early stage and subsequent damage results from the rubbing of the wear particles against the surface.

The effect of fretting corrosion on the fatigue strength was studied. Fenner and Field (2.5-18) showed that the adverse effect results predominantly from the first stage of fretting, involving the welding and fracture of metal to metal surfaces. The explanation of both fretting and corrosion effects was made by Forrest (2.5-19) in the way that fatigue cracks are formed at an early stage and most of the life is associated with crack propagation.

The resistance to fatigue failure under fretting was shown to increase by surface treatments such as shot-peening, surface rolling or by the use of surface coatings (2.5-20). The considerable improvements that can be obtained with an interference fit was demonstrated by Law (2.5-21).

With the aid of tight clamping in a bolted joint a considerable proportion of the load can be transmitted by friction between the plates, particularly in thin sheet. Fisher and Winkworth (2.5-22) obtained an increase in fatigue strength of 4 times by tight clamping of the bolts on 1.5 mm thin aluminium alloy sheet. Heywood (2.5-23) showed that it is advantageous to combine tight clamping with a very close spacing of the bolts.

Mechanism of Fatigue Failure

A better understanding of the processes underlying fatigue failure began gradually to emerge from the development of improved and new techniques of optical and electron microscopy. Forsyth (2.5-24) and Thompson (2.5-25) showed that fatigue stressing resulted in the extrusion of thin ribbons of metal from the surface. The effect was subsequently observed in other metals, the size of the ribbons depending on the material and stress conditions. It was also found that sharp crevices or intrusions were formed at the surface as a result of fatigue stressing. The most significant outcome of the work with improved techniques was, without doubt, the detection of microfatigue cracks at an early stage in the life of the specimen. This was demonstrated by Thompson (2.5-26) for copper and has been observed subsequently on a wide range of materials. The regular way in which fatigue cracks spread was shown by Forsyth and Ryder (2.5-27) by examination of fatigue fracture surface with a high power optical microscope. They showed that one striation is produced by each cycle of stress, subjecting an aluminium alloy wing spar to a programmed fatigue test.

Industrial Significance

Much of the impetus for research on fatigue since the Second World War has come from the realization of its dangers in aircraft structures. One outcome of this interest was the introduction of Data Sheets on Fatigue by the Royal Aeronautical Society, which has covered a wide range of applications.

Aircraft designers started to direct more attention to the rate of propagation of fatigue cracks and also to the static strength of cracked structures. It was found that the effect of a crack on the static strength was dependent on the ductility of the material. Although the strength of mild steel was scarcely affected, fracture in aluminium alloys could occur at a stress, based on the uncracked area, of only half the tensile strength (2.5-28). In tests on full-scale aircraft wings of the aluminium alloy 24 S-T, it was found that with 10 percent of the tension area cracked, the static strength was reduced by 40 percent (2.5-29).

Another branch of engineering in which fatigue research was actively carried out was welding. In 1960, a conference was held in Cambridge on the fatigue strength of welded structures. Appreciable improvements were obtained in the fatigue strength of welded structures by inducing beneficial residual stresses (2.5-30).

Until the 1960s, much of the emphasis in metallographic research was directed mainly at crack initiation. However the discovery that microfatigue cracks are formed at an early stage in the fatigue life led to the conclusion that a study of the rate of crack propagation should be very important in the fatigue process. It then seemed certain that there would be an increased interest in crack propagation, both on a macro-scale and on a micro-scale. On the other hand, there started an increasing practical interest arising from the greater use of design for a finite life. The risk of fatigue cracks was accepted, provided that they could be prevented from causing complete failure.

2.6 Fatigue Research since 1960

During the last two decades research has increasingly concentrated on the subject of crack growth which centres around the whole field of fracture mechanics. By the application of fracture mechanics concepts, an analytical approach to crack propagation can be obtained and this is very important in the fail-safe approach to fatigue design. The most important aspect of the use of fracture mechanics is the single-valued correlation coefficient in the linear elastic range between the stress intensity factor, K, and the rate of fatigue crack growth $\frac{\Delta a}{\Delta N}$, where a is the crack length and N is the number of cycles. The stress intensity factor, K, is related to the stress concentration factor, K_t, through the following definition of the stress intensity factor:

$$K = \lim_{\rho > 0} K_t \, \sigma \, \frac{\sqrt{\pi\rho}}{2} \qquad (5)$$

with K_t and σ based upon the gross cross-sectional area, and ρ is the tip radius. For the case of a central slit in a sheet specimen subjected to tensile loading at right angles to the slit the expression for K (with the crack length much smaller than the specimen with) becomes:

$$K = \sigma \sqrt{\pi a} \qquad (6)$$

In general the stress intensity will be of this form, but modified to account for particular geometry, in which case it can be expressed as:

$$K = \sigma \sqrt{\alpha \pi a} \qquad (7)$$

where α is a factor which takes into account the particular geometry. Consideration is recently given to the structural modifications such as stringers on the rate of crack growth; however the bulk of research in the past has involved specimens of constant thickness in which material response rather than structural response can be established (2.6-1). The characteristic dependence of the rate of fatigue crack growth on the stress intensity factor is shown in the following figure.

There are two asymptotic limits to the curve. The upper limit is set by the fracture toughness of the material, K_c. The lower limit is referred to as the threshold for crack growth, K_{th}. This latter quantity has recently been of considerable interest, for a new design philosophy based upon the quantity is emerging. Many parts before going into service already contain crack-like defects as in the case of welded joints. Some of these parts may have to be designed for infinite safe life. In the absence of defects this would entail designing at stresses based on the 10^8 life of the S-N curve, but in the presence of defects the new approach is to insure that the stress intensity associated with defects is kept below the threshold level for crack growth. The rate of fatigue crack propagation can be expressed as a function of the stress intensity factor, but to put such an expression on a rational basis, consideration should be

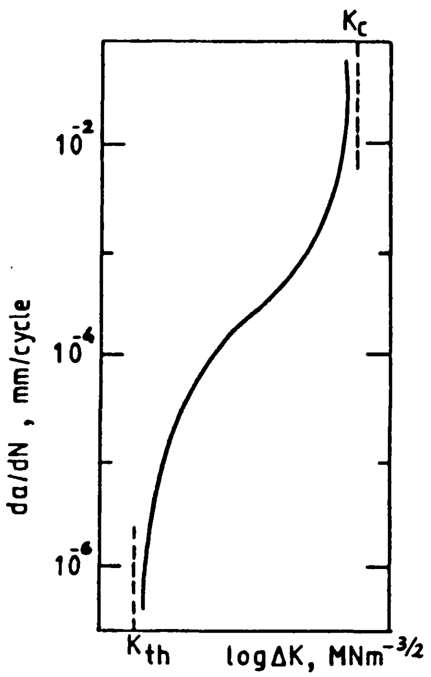

FIG.2.2 A characteristic curve of crack growth rate against stress intensity range [Ref.(2.6-1)]

given to the modes of separation involved. At low crack growth rates i.e. below 25.10^{-4} mm/c at R = O (R being the ratio of minimum to maximum stress in a simple loading cycle), a ductile mode of crack advance associated with plastic blunting on loading, and tip sharpening on unloading is thought to be dominant (2.6-2). At higher rates of crack growth where the peak stress intensity approaches the fracture toughness, static modes of separation such as ductile rupture become operative and accelerate the rate of crack growth.

Investigations are continuing to study the effect of various parameters on fatigue strength, the aim is to predict by theoretical means from basic material data the strain distributions in components where macroplasticity is involved; to relate strain distributions to crack propagation rates in both high cycle and low cycle regions and to study the significance of energy changes in fracture mechanics.

2.7 Concluding Remarks

Fatigue has now been the subject of research investigations for well over one hundred years, and despite the progress made, failures continue to occur. This situation is due in part to the complex nature of the fatigue process and the stress-material-environmental interactions involved therein. However much of the fatigue problem is simply due to poor communications. Many designers do not know how to deal properly with fatigue and do not treat it as a matter of concern until product failures start to occur.

In recent years, improvements in analysis rather than improvements in materials resistance to fatigue have been obtained. While it is to be hoped that material improvements will be forthcoming, thus far this goal has been a difficult one to obtain. Perhaps more modest goals such as reduction in scatter, in notch sensitivity, and in environmental sensitivity may be more realizable and useful than improvements in average properties. At any rate, a wider dissemination of information already available about fatigue design would constitute a positive step toward the elimination of fatigue failures.

Today, there are basically two main approaches to design against fatigue. In dealing with large sheet structures such as an aircraft wing which may contain cracks of significant size, application of fracture mechanics approach is generally followed. Such structures are aimed to be fail-safe so that in the event that fatigue cracks do develop their presence will not be catastrophic. On the other hand, there are numerous components in every field of engineering that are to be designed for a safe-life. The S-N properties of the material provide the bases for these components.

During the survey of fatigue literature the following remarks made by Heywood (2.7-1) on the rolled bolts has drawn attention and led to this research work; 'no results appear to be available to show the effect of mean stress on bolts having rolled threads'. The fatigue life to failure is essential in investigating the fatigue characteristics of these bolts in tension and this must be determined experimentally.

The designer will be able to assess his problem and find the correct solution if he has available the complete fatigue life characteristics of his component or joint. This knowledge will include the mean and alternating loading and the confidence limits for his results. This work aims to demonstrate in this regard the information required for tension bolts.

3. SELECTION AND SPECIFICATION OF BOLTS AND SCREWED BARS

3.1 Introduction

As the primary purpose of this research was to study the characteristics of bolts in tension, it was not necessary to choose any particular grade. It was therefore decided to choose bolts that were readily available from the local supplier. This enabled the bolts and nuts to be purchased in boxes of 50 and these can reasonably be assumed to be a typical batch from continuous production. The size of the bolts was determined by the capacity of the fatigue machines and this led to the use of 12 mm diameter for the Avery Machine and 10 mm diameter for the Amsler Vibraphore. The choice was further conditioned by the design of the machine adapters for loading the bolts. (See paragraph on Test Adapters.) In particular the use of moderate strength test bolts reduced the fatigue design problems in the adapters.

On the other hand the choice of commercial grade steel bolts had a direct advantage in the investigation of the scatter distribution of the fatigue life. It is known that high grade aircraft quality bolts are manufactured to close quality control and therefore the standard deviation would be reduced making a detail study more difficult without altering the principles involved.

3.2 Specification of the bolts

The bolts are generally specified by BS 4190:1967 (3.2-1) and it was not considered possible to obtain from the manufacturers any further details of their standards. In common with other British Standards BS 4190 has been written to provide a standard from the users application and not to control the manufacturing methods. The specification therefore concludes all dimensions in relation to tolerances and finish. It also gives the required range of mechanical strength of the finished bolt without reference to the material specification. Three material grades are referred to and tensile strength and hardness values are given. Details of the geometry of bolts are shown in Figure 3.1.

3.3 Properties of the steel bolts

A sample of the bolts and nuts were used to find the actual geometric properties and mechanical strength properties. The results of the measurements are given in Table 3.1 and these are generally the mean values of a number of measurements. The M-10 bolts supplied by G.K.N. were in boxes with the BS 4190 specification and properties do meet that specification with regard to geometry. The specification for strength and hardness gives minimum and maximum values and the test bolts exceed the properties quoted as maximum values for grade 4.6 designation. The M-12 bolts were also ordered as low grade 4.6 but no BS number was quoted on the box. The results of the tests given in Table 3.1 shows that the strength and hardness considerably exceeds BS 4190 grade 6.9 properties. The mean test properties are therefore used in this thesis as a basis for strength reference and not the specification.

Table 3.3 gives the statistical deviation of the strength properties. The standard deviation on strength and hardness is quite low as would be expected from quantity manufacture. The coefficient of variance is between 2 and 3% and therefore if a safe design strength is taken as three deviations below mean the reduction is only 6% to 9% and the design strength is well above specification.

3.4 Properties of the screwed bar

There were no standards with which to compare the screwed bar data given in Table 3.2. The mean values of the strength/hardness properties were measured along with all geometric dimensions and these are recorded in the table. The strength properties for commercial mild steel are not usually specified but comparison with BS 970, 'Properties of low carbon steels' show them to be well above the lower grades. Details of the geometry of the screwed bar are shown in Figure 3.2.

3.5 Microscopic Evaluation of Specimens

3.5.1 Preparation

Samples along the lines of longitudinal and transverse sections were cut from both bolts of 10 mm and 12 mm dia., as well as of screwed bar of 40.2 kN ultimate tensile strength. They were ground on successively finer grades of wet and dry paper ensuring washing between each different paper. The final preparation on diamond lapping wheels was performed to produce a stratch condition of less than 1 μ. Then the surfaces of the samples were etched in 2% nitric acid in methanol.

3.5.2 Inspection

The transverse surfaces of the 10 mm bolt and 12 mm screwed bar samples, as microscopically evaluated, display the same pattern of commercial steel in the annealed condition. The microstructures comprise equiaxed-grains of ferrite and pearlite. The proportion of pearlite grains to ferrite grains is about 10% suggesting an optical carbon assessment of .1%C. The grain size of the screwed bar is considerably greater than that of the bolt. Samples have a low inclusion count and there is random inclusions of manganese sulphide. The material has not been decarburized (Figure 3.3).

The longitudinal sections of the same samples are similar to the transverse sections except there are some areas of pearlite segregation and pearlite and phosphorus banding. There are indications of cold rolling and these are the stretching out manganese sulphide and some complex silicade inclusions longitudinally. Also grains have been deformed due to cold rolling indicated by the flow of material around the root of the thread (as shown in Figure 3.4)

Samples of 12 mm bolt displays a microstructure of acicular martensite. There is little evidence of spheroidized-carbide precipitating out from the martensite and consequently the material has experienced very little, if any, tempering (Figure 3.5).

Examination of the longitudinal section confirmed the material to be homogeneous with little evidence of banding of segregation as would have arisen during rolling. Material was heated up above A3 temperature on the Fe-C equilibrium diagram to the fully austenitic condition and quenched to form a low carbon martensite (Figure 3.6).

4. DESCRIPTION OF TEST EQUIPMENT

4.1 Introduction

Fatigue tests of both screwed bars and bolts were carried out on two different axial fatigue machines; a modified Avery-Schenck Pulsator of 6 ton capacity and an Amsler 10 ton Vibraphore. A Denison Tensile Testing Machine which is of 10 ton capacity was used for tensile tests and for calibration of load cells.

In order to attach the specimens onto the machines, four different test adapters of high strength steel were designed.

4.2 Avery-Schenck Pulsator

This is a resonant frequency machine with two large coil springs, one inside the other (see Figure 4.1). The static load is applied to one end of the specimen by means of a d.c. motor driving a lead screw on the one end of the inner spring. Another d.c. motor drives an out of balance mass which causes the larger outer spring to vibrate and apply the dynamic load. The specimen is attached between this outer spring on the one end and the dynamometer on the other end by means of wedge grips. In this way the specimen and the spring are connected in series. However since the spring is much more flexible than the specimen natural frequency of the machine can hardly be affected by the stiffness of the specimen. This is an important feature of this machine because after a crack starts to propagate the machine still maintains the same dynamic load as before. This is not the same with Vibraphore where as soon as a crack starts propagating the machine loses resonance and the load reduces very rapidly.

The modified version of the machine is equipped with an electronic feed-back control system and a digital display unit of mean and alternating loads. The feed-back system provides precise control of the applied alternating load and it greatly facilitates exact setting of the static mean load. The alternating load cycles are recorded on a revolution counter which is driven by a flexible drive from the d.c. motor applying dynamic load. The machine is equipped with a cut-out which switches the machine off when the specimen breaks and the cycle counter can then be recorded at the next opportunity. The machine was calibrated previous to testing by means of a load cell which was itself calibrated on the Denison Tensile Testing Machine.

4.3 Amsler Vibraphore

The Amsler-Vibraphore is driven electromagnetically and it operates at the resonant frequency of the mass/spring components as shown in Figure 4.2. The electromagnet excites the main moving mass of several weights which are attached through grips at the upper end of the specimen. The lower end of the specimen is connected through the dynamometer to a much heavier opposing mass of relatively 'infinite' magnitude. The dynamometer measures strains and hence the loads on the specimen. Mean stress is applied by means of a low stiffness static spring arrangement. The frequency can be varied mainly by altering the weights or to a lesser extent by altering the stiffness of the test piece. A decrease in the operating frequency is obtained by increasing the weights thus a range of 50 Hz to 300 Hz can be attained.

This machine has recently been equipped with a 'solid state vibraphore conversion electronics' unit. With this new unit the operation has been substantially changed in that the power amplifier constantly drives the mechanical resonant system in a sinusoidal fashion rather than feeding discreet pulses of energy as in the original Amsler-Vibraphore system. While providing a tighter control over the machine the new system does not require any tuning during loading or operation. All power compensation is fully automatic. The applied load in the system is measured electronically thus eliminating the old mirror/optical system. A digital display module with ranges of 1:1 and 10:1 monitors at the push of a button, the maximum tensile load, maximum compressive load, mean load, dynamic load and frequency all to $\pm 1\%$. Each 100 cycles are counted up to a maximum of 10^8 cycles and the counter can be manually pre-set to shut the machine down at a predetermined number of cycles.

4.4 Avery-Schenck Test Adapters

In order to attach the test specimens to this machine two different adapters were designed. These were made of high strength steel. One of them containing an inner screw was used to test the screwed bars loaded directly, i.e. without any nuts attached, as shown in Figure 4.3. The other one was used both for screwed bars loaded through nuts and for 12 mm bolts, as shown in Figures 4.4 and 4.5a, 4.5b.

4.5 Amsler-Vibraphore Adapters

Two new adapters were designed to fix the specimens onto the Vibraphore. The one designed for the screwed bars tested on this machine was similar to its counterpart on Avery-Schenck, see Figure 4.6. The adapter for the 10 mm bolts was designed to be two identical pieces machined from solid steel round bar. It was found essential for smooth running of the vibraphore to avoid joints with associated damping in the adapters. The design is shown in Figure 4.7.

5. FATIGUE TESTS ON SCREWED BAR

5.1 Early tests covering a range of variables

As an initial approach to the assessment of the fatigue properties of screw threads in bolts it was decided to study the thread alone by using screwed bar material. Commercially available screwed bars of 1 m length were in stock. By cutting equal pieces, each 120 mm long, the consistency was maintained in the specimen material as they were used in a random order. Static tensile specimens could also be taken from among the fatigue specimens to help in the statistical understanding of the basic material properties.

Two alternative methods were chosen to load the screwed bar in tension. The first was to screw the specimen ends into screwed blocks held in the grips of the fatigue machine as shown in Figures 5.1 and 5.2, these are referred to as 'direct loading'. The second method was considered more relevant to the main topic of bolt fatigue and the load applied through nuts in the end fittings shown in Figure 4.4. This method involved transferring the load through a nut at each end of the screwed bar and is equivalent to the nut/thread end of a tension bolt.

A large number of tests were done on the 12 mm dia. screwed bars and all the results are collected together in Figure 5.3. There are several variables involved which are separated in the following Figures 5.4 and 5.5, but it is of value to note that the scatter is still not excessive. A mean fatigue life has been produced from the equation shown alongside the curve in Figure 5.5. A mean load of 15 kN was assumed in evaluating this equation for it was meant to represent the average fatigue life of all the specimens. The curve of the mean fatigue life fits the pattern well. From the static test results of Table 3.2 the mean strength of the second batch is used in the fatigue equation and two lines giving two standard deviation limits in terms of static strength are superimposed. It is seen that the majority of results lie within these limits and those lying outside the boundaries can be explained in terms of the variables in testing the specimens.

The tests made at 10 kN mean load are shown in Figure 5.4 and in each case the loading was direct through screwed end fittings. The two groups of Vibraphore results are indistinguishable but there is a suggestion that the Avery machine gives a shorter fatigue life than the Vibraphore, this trend has been noted in other research using these machines. Although the higher strength material in the Avery gives results comparable with the Vibraphore tests the main batch Avery results have a seemingly shorter life over most of the range. While these trends exist the collective results form a good fatigue curve with quite a low level of scatter.

The tests done at 20 kN mean load are brought together in Figure 5.5 and each individual group is seen to give quite a good fatigue curve of the same form as that given by the equation in the figure.

5.2 Fatigue tests in the Avery to study scatter distribution

The tests covering a range of variables established some general trends which led to a choice of carefully controlled tests to investigate particular variables. Some of the existing tests on the Avery-Schenck at 10 kN mean load were therefore selected and additional tests were carried out at each alternating stress to give the results shown in Figure 5.6.

The fatigue test results have been analysed by the method given in chapter 9. The computer program uses the results as input data and from this the graph shown in Figure 5.6 is plotted by the computer. In this figure the 95% confidence limits are also given for each level as well as the equation to the curve.

An alternative method of plotting the same results using the computer is shown in Figure 5.7. In this case only the mean fatigue life of the three results at each stress level is plotted and it is seen to compare well with the same theoretical curve.

This method of testing and analysis has been repeated for a mean load of 20 kN and the results are shown in Figures 5.8 and 5.9. In this case, some individual test results between successive levels where three tests were performed are also added on to the graph to indicate the relative positions of such tests. Again the theoretical equation is given and it is seen to fit the experimental results quite well. All the results are shown in tables 5.1 to 5.6.

5.3 Fatigue tests in the Vibraphore to study scatter distribution

A series of new tests was carried out to study the scatter in the fatigue life of screwed bars in the Vibraphore. These tests were designed similar to those performed in the Avery. Again two mean load levels, namely 10 kN and 20 kN, were maintained and three tests were done at each selected alternating stress level. Figure 5.10 shows test results as plotted by computer for 10 kN mean load. Every group together with its confidence limits ($\gamma = 0.95$) is shown in the figure as well as individual test points between successive groups along the best-fitting Jefferson's Curve. Isolated tests between successive groups were designed to investigate the proximity of such random test points to the theoretical curve which was meant to pass through the means of three test groups. The relative positions of mean values with respect to the theoretical curve are shown in Figure 5.11 together with the isolated test points.

The same procedure was repeated for 20 kN mean load. Figure 5.12 shows the results as groups with confidence limits while in Figure 5.13 the mean values are located. The theoretical curve and individual test points are also shown in the figures. The equation of each curve is given alongside. Both of the theoretical curves plotted are seen to fit the experimental results very well.

5.4 Comparison of the Avery-Schenck and Vibraphore results

Though reasonable fits have been obtained by the Avery-Schenck test results, those from the Vibraphore appear to have comparatively better fits. The theoretical curves of the Vibraphore tests are everywhere within the 95% confidence limits. This is not the case with the theoretical curves of the Avery-Schenck test results. One group of tests lie to the left of the curve at 10 kN mean load. Exactly the opposite holds true for the curve of 20 kN mean load. To some extent this discrepancy can be accounted for by the slight lack of smooth operation of the Avery-Schenck test machine at a few relatively high load levels. On the other hand, the confidence limits belong to the mean of the test results at each alternating stress level. Hence the equation represents a 50% probability curve at a confidence level of $\gamma = 0.95$. If the confidence level is increased the confidence interval will open to probably include the curve within the new limits. So the limits will appear to be more meaningful if the closeness of the mean to the theoretical curve, associated with these limits, are also considered.

From this standpoint the mean value of each group lying either side of the curve is quite near to the curve. In general, the screwed bars tested in the Vibraphore appear to have longer fatigue lives than those tested in the Avery. This distinction may not be seen clearly if only the values of the equation constants of corresponding curves are compared. In fact, the equation constant of the curve of test data from the Avery is nearly 20% higher than the corresponding Vibraphore curve at both mean stress levels. This result if considered alone can lead to the false impression that Avery tests have the longer fatigue lives, but if the effect of the endurance limit, S_e, upon the alternating stress, S_a, is also taken into account the difference can be evaluated. On the other hand, it is worth noting that doubling the mean load on each machine has caused the constant B to increase to a value approximately twice the original value.

5.5 Effect of mean stress on the endurance limit of screwed bar

Tests to investigate the effect of the mean stress on the endurance limit were carried out in the Vibraphore. The endurance stress at 10^7 cycles was determined at some selected mean stress levels with regular intervals. The endurance stress curve plotted is shown in Figure 5.14. This curve closely resembles the Gerber parabola. This demonstrates that the endurance limit of the screwed bar appears to have little dependence upon the mean stress for the range where $S_m < 0.5\ S_t$. However over the range $S_m < 0.5\ S_t$ the Gerber equation is nearly constant with mean stress and the same trend has been noticed previously as one of the characteristics of the Jefferson's equation. If it is considered that in design practice mean stress is usually held in the above range, Jefferson's curve can safely be adopted for design purposes covering that range.

6. FATIGUE TESTS ON BOLTS

6.1 Introduction

Following on the experience gained in testing screwed bar specimens, tests were made on bolts using partly same fittings. In the case of bolts the load must necessarily be applied through the bolt head and the nut respectively. The first series of tests were carried out in the Avery-Schenck using 12 mm bolts which were consistent with the adapters. The second series of tests done in the Vibraphore were performed on 10 mm bolts and nuts.

The bolts and nuts were obtained in boxes of 50 to preserve consistency as much as possible and over 200 bolts were tested. At each mean load tests were made at many stress levels to provide a close coverage of the curve of alternating stress against fatigue cycles to failure. To provide data for the statistical analysis of Chapter 9, tests were repeated at each stress level to give generally between 3 and 7 results.

The test results were typed into the multics computer program and were plotted by the computer to give the graphical results. Given the optimum equation constants the program then prints the theoretical curve shown on each figure. The program also locates the 95% confidence limits and as an alternative figure the statistical mean fatigue lives.

Considering the argument explained in Section 9.3.3 (d) the P-S-N curves were calculated and drawn on log-log scale for 10 mm bolts. A confidence level of γ = 0.90 was chosen and with three probability percents, 10%, 50%, 90% best curves were plotted at three mean stresses by the least square method.

6.2 Fatigue tests in the Avery-Schenck machine

The tensile strength properties of the 12 mm bolts used in these tests are given in Table 3.3 and from these it was decided to concentrate a large number of tests at the one mean load level of 20 kN. Three tests were made at each of 13 alternating stress levels and the use of quite a high level of mean load allowed tests to be made at high alternating loads, thus providing results over a wide range of fatigue life. The tests were made using the adapter fittings shown in Figures 4.5a and 4.5b and as there was a free length of bolt between the fittings it was not possible to take compression, that is the alternating load must always be less than the mean load.

The test results were plotted by the computer and are shown in Figure 6.1 along with the 95% confidence limits and the computer fitted theoretical curve. The theoretical equation is given in the figure incorporating the stresses relevant to the test conditions and the chosen constants.

The test results at each stress level are seen to give a consistent and low scatter and generally the 95% confidence limits are around the value of 2 : 1 on fatigue life. The range naturally increases at the lower stresses toward the endurance stress level. In this presentation of the results there appears to be more scatter about the theoretical curve at particular alternating stress levels thus demonstrating the necessity of making tests at a reasonable number of stress levels.

The alternative presentation of the same results in Figure 6.2 plots only the mean value to the tests at each alternating stress level. Although the same trend is seen as in Figure 6.1 this presentation does give confidence to the use of the theoretical curve as a representative of the mean fatigue life.

6.3 The effect of mean load on the fatigue life

The second series of tension fatigue tests were done on the Vibraphore machine using 10 mm bolts and nuts. The pattern of testing and processing of the results is the same as described for the Avery machine in Section 6.2. This series of tests is more extensive in that the tests are repeated for 3 levels of mean load chosen at 10 kN, 15 kN and 20 kN. The possible range of alternating load is limited in relation to the mean load because the maximum load must be less than the bolt tensile strength and the minimum load must be reasonably above zero tension. The individual fatigue test results and the confidence limits for the three mean loads are given in Figures 6.3 to 6.5 and they show a similar pattern of scatter to the results obtained in the Avery. At the 10 kN mean level shown on

Figure 6.3 the scatter between levels of alternating stress is rather worse than the scatter within each level. In this particular case the theoretical line is not necessarily the best curve. However with increasing mean load the theoretical curve fits better and the agreement on Figure 6.5 is good.

The mean result of the tests at each alternating stress level is given for the three mean load levels in Figures 6.6 to 6.8. These results give good support for the theoretical line particularly at the two higher mean loads. The higher mean loads have the advantage of allowing full coverage of alternating stress range, which is not possible at 10 kN as the top of the curve represents compressive loads in the minimum part of the cycle.

The relationship between the results at the three mean loads is good and will be discussed in Chapter 10.

6.4 The influence of mean stress on the fatigue limit

The fatigue test program on 10 mm bolts using the Vibraphore machine was extended to study the influence of mean stress on the fatigue limit. Fatigue limit is defined as the stress which gives a life just exceeding 10^7 cycles, a definition which is often accepted. The test results are given in Table 6.1 and the method used was, after examining the data of theoretical curves, to make several tests at the chosen mean load level to find the highest stress for which the bolt could remain unbroken at 10^7 cycles. For a range of mean load levels the alternating stress which meets the requirement is indicated in the table.

The fatigue limit curve which was produced from this test method is plotted in Figure 6.9. It is seen that the fatigue limit stress is largely independent of the applied mean stress. This behaviour of the bolts was noticed by previous researchers (6.4-1).

At the higher mean stresses the maximum tensile stress in the bolt approaches its tensile strength and the allowable alternating stress decreases rapidly.

6.5 Comparison of the bolt tests on the Avery-Schenck and Vibraphore

Both the Avery and Vibraphore results agree with their respective theoretical curves quite well, and deviations from this behaviour are usually towards the lower limit of the confidence interval. As with the screwed bars one group of bolt tests is an exception to this general trend in the Avery-Schenck. At this point of tensile stress, which is about 425 N/mm^2, the machine was producing excessive alternating load which could be expected to reduce the fatigue life as shown in Figure 6.1.

Although the tensile strength of 12 mm bolt is considerably higher than the 10 mm bolt they both appear to have the same fatigue limit at 10^7 cycles. Avery test results have shown slightly shorter fatigue lives than those obtained from the Vibraphore. However, in the case of 12 mm bolt the shorter fatigue lives can to a great extent be accounted for by the microstructure of this bolt type. As shown in Figure 3.5 the microstructure of the 12 mm bolt is martensitic and any trace of tempering can hardly be detected. Microstructure is known to have an important role in fatigue and untempered martensite is given as the one with the lowest ratio of endurance limit to tensile strength among the main microstructure types of steel (6.5-1).

6.6 Probability fatigue curves of the bolts

The theoretical curves plotted for the bolts are the mean curves representing 50% probability of survival and the confidence interval with $\gamma = 0.95$ is given for the mean of each test group at every alternating stress level. In order to examine the effect of probability attached to the fatigue lives of bolts and after giving due consideration to the argument explained in detail in section 9.3.3 (d) it was decided to calculate and plot 10%, 50% and 90% survival curves of the 10 mm bolts with a confidence of $\gamma = 0.90$. To this end only those groups consisted of 3 or more tests were taken into account for three mean load levels of 10 kN, 15 kN and 20 kN. The results are shown in Figures 9.3a, 9.3b and 9.3c plotted on log-log paper. And the survival curves are not forming parallel lines but tend to diverge as they approach the endurance limit. This situation accrues from the fact that scatter increases as the endurance limit is approached. At any stress level covered by the lines test specimens would be between the P = 90 life and P = 10 life, 80 percent of the time with only 20 percent falling on either side of the range at a confidence level of $\gamma = 0.90$. This kind of analysis and representation of test data, which is used quite often, gives satisfactory results on a log-log plot only in the region where the S-N curve assumes an approximately straight line. However, this kind of plot fails to give any estimate about endurance limit and suggests a finite fatigue life right down to zero alternating stress. The results excluding those of the P-S-N curves are recorded in the tables 6.2 to 6.6.

7. BASIC FATIGUE RELATIONSHIPS

7.1 Fatigue Limit

In the course of historical development of fatigue phenomenon consideration given to the fatigue behaviour of materials has been changed to follow a line more or less parallel to the needs of the designer. At first, in the belief that design was based entirely on a definite fatigue limit, the main objective was to determine this value experimentally. Also in the desire to express this limit in terms of one or two basic mechanical properties such as ultimate strength and/or yield strength, many different formulae were proposed. The range of application and the degree of reliability of these equations are no doubt limited by various factors such as type of loading, surface conditions, material properties, etc. Most of the equations of this kind, still widely used today, have been based on the rotating fatigue test data of smooth round-bar specimens. Resulting from the very nature of this loading type stresses are alternating with equal tension and compression amplitudes around a zero mean. When these equations are to be applied to the similar machine parts of the same material under different conditions a set of correction factors should be introduced to account for the differences in loading condition, surface finish, size, etc. The following are but a few examples of this kind of equation currently in use:

$S_e = 0.454\ S_t + 8.4$ (N/mm^2) for normalized carbon steels (340-700 N/mm^2)

$S_e = 0.515\ S_t - 24$ (N/mm^2) for heat-treated carbon steels (400-1300 N/mm^2)

$S_e = 0.383\ S_t + 94$ (N/mm^2) for heat-treated alloy steels (800-1300 N/mm^2)

$S_e = 0.484\ S_t$ (N/mm^2) for stainless steels (500-1300 N/mm^2)

where S_t is the tensile strength of steel, S_e is the fatigue limit at 10^7 cycles. The above formulae are more preferable than the well-known assumption that for wrought steels $S_e \cong 0.5\ S_t$ because they have been derived considering the microstructure of the steel in question (7.1-1).

7.2 Mean Stress Effect

Taking into consideration the fact that many machine and structural parts are subjected to alternating loads/stresses superimposed by some static loads/stresses substantial investigation of the effect of tensile mean stress on long-life fatigue strength has been made. Although the general trend indicating that tensile mean stresses are detrimental is quite evident considerable scatter exists. The earliest attempt to express the variation of the alternating stress or the fatigue limit as a function of the mean stress was that of Goodman (7.2-1). He assumed that between the limiting values of mean stress $S_m = 0$ and $S_m = S_t$, the safely applied alternating stress amplitude decreased linearly. Gerber (7.2-2), on the other hand, suggested that between those two limiting values of mean stress, decrease in alternating stress followed a parabolic law. Two additional relationships were later formulated by replacing S_t with S_y (Soderberg line (7.2-3)) and S_t with σ_f (Morrow line (7.2-4)) where S_y and σ_f are the tensile yield strength and true fracture stress, respectively.

The following equations represent these tensile mean stress effects:

Modified Goodman
$$\frac{S_a}{S_{a_o}} + \frac{S_m}{S_t} = 1 \qquad (7.1)$$

Gerber
$$\frac{S_a}{S_{a_o}} + \left(\frac{S_m}{S_t}\right)^2 = 1 \qquad (7.2)$$

Soderberg
$$\frac{S_a}{S_{a_o}} + \frac{S_m}{S_y} = 1 \qquad (7.3)$$

Morrow
$$\frac{S_a}{S_{a_o}} + \frac{S_m}{\sigma_f} = 1 \qquad (7.4)$$

where S_a and S_{a_o} are the alternating stress at some mean stress and alternating stress at zero mean stress respectively and S_t is the tensile strength. All four equations have been used in fatigue design when modified for notches, size, surface finish and environmental effects.

Investigations made by using either direct stresses or torsion stresses have shown that metals of different size, geometry and structure vary in their behaviour when a static stress is superimposed on the alternating stress. Some maintain a fatigue range almost constant and to some extent independent of the static stress. For others, the safe range of alternating stress begins to fall as soon as any static stress is applied. Although such variations can be accounted for by a number of reasons generally two important factors must always be taken into consideration in this regard for notched parts.

Firstly, mean stresses inherent in the unloaded part are often much greater than the mean stresses caused by external loads in service or superimposed during testing. Secondly, as long as the maximum tensile stress at the notch root of a part does not increase appreciably beyond the yield stress of the material until the plastic deformation extends over a large area of the specimen cross-section, the alternating stress that can be applied safely will remain nearly constant along the range of mean stress covering general design practice (7.2-5). On the other hand, there is no doubt about it that compressive mean stresses cause increase of up to 50 percent in the alternating fatigue strength. This point is sometimes overlooked since compressive residual stresses can produce similar beneficial behaviour (7.2-6).

7.3 S-N equations

Analytical equations relating fatigue life of a part with the applied stresses are significant and useful for a number of reasons. Whenever the design is to be based on finite life the entire S-N curve becomes important. Often the limiting alternating stress can be evaluated quite correctly by means of a curve represented by such an equation. In fatigue testing they play an important role as a guide in determining the number of specimens to be tested and in analysing the results from different viewpoints. The fatigue curves obtained from different testing machines such as rotating bending, push-pull and alternating bending types are generally different for the same material.

The fatigue curve for a particular material is highest for specimens subjected to alternating bending and is lowest for those specimens subjected to uniform alternating stresses as being the case in a push-pull test. Such differences can be traced to two main causes:

(a) Differences in presentation of the results

(b) Differences due to the fatigue process itself

Very recently, Esin (7.3-1) has developed a method for correlating the fatigue curves obtained by different methods of testing. This method which has been experimentally verified is based on the micro-plastic strain energy criterion, of which Esin and Jones developed a mathematical model in an earlier work (7.3-2).

An S-N equation put forward by Weibull (7.3-3) is as follows:

$$(S - E) \quad (N + B)^m = A \qquad (7.5)$$

where S is the alternating stress, E is the endurance limit, N is the life cycles and A, B, m are constants. In addition to the difficulty encountered in evaluating its constants Weibull's equation does not include the mean stress among its parameters. Throughout this research work a new equation has been used which will be explained in the next chapter.

8. S-N EQUATION PROPOSED BY JEFFERSON

During research at the University of Bath, Dr Jefferson (8.1-1) has developed the following equation:

$$S_a = \frac{S_e}{1 - \left(1 - \frac{S_e}{S_t - S_m}\right) \frac{\log B}{\log (N + B)}} \qquad (8.1)$$

where the following notation is used:

S_a is the alternating stress $\left(= \frac{S_{max} - S_{min}}{2} \right)$

S_e is the fatigue limit as $N \to \infty$

S_t is the ultimate tensile strength of the material

S_m is the mean stress

N is the cycles to failure

B is an equation constant

This equation exhibits the general characteristic shape of the S-N curve. It can be seen from the equation that as N approaches to infinity the alternating stress is equal to a constant fatigue limit and at the beginning of the cycles it is equal to $(S_t - S_m)$. It also satisfies the condition that as the mean stress approaches the ultimate for the material the alternating stress, S_a, about the mean must reduce to zero. The proposed equation has also been fitted satisfactorily to the mean curve of the R.Ac.S. data on dry lap joints (8.1-2).

By examining a family of curves drawn at different stress levels with arbitrarily chosen values of parameters as shown in Figure 8.1 the following can be deduced:

The curves with constant mean stress converge rapidly towards a common endurance in the range covering the middle life and the effect of mean stress on the alternating stress is not appreciable throughout the high cycle range. On the other hand, thc constant B plays a different role as shown in Figure 8.2. An increase in this constant causes the middle life portion of the curve to move along the horizontal life axis by an amount corresponding roughly to the increase of B. Since the life axis is logarithmic an increase in the value of B for a range covering low cycles will be more pronounced than that produced by the same number of cycles in the middle and high cycle ranges. Contrary to the effect of B the endurance stress affects considerably the shape of the curve in the high cycle range but becomes less effective at lower fatigue lives as shown in Figure 8.3.

Jefferson's equation may be rearranged to obtain an expression which relates the alternating stress and mean stress at a constant life.

Denoting $\dfrac{\log B}{\log (N+B)} = C$ a constant it becomes

$$S_a = \frac{S_e}{1 - \left\{1 - \dfrac{S_e}{S_t - S_m}\right\} C}$$ and rearranging

$$S_a - S_e = S_a \left\{1 - \frac{S_e}{S_t - S_m}\right\} C$$

$$(S_t - S_m) \quad (S_a - S_e) = S_a.C. \, (S_t - S_m - S_e) \qquad (8.2)$$

In the equation (8.2) if S_a is replaced by S_{a_o}, the alternating stress limit at a zero mean, the equation becomes:

$$S_t \, (S_{a_o} - S_e) = S_{a_o}.C \, (S_t - S_e) \qquad (8.3)$$

Dividing equation (8.2) by equation (8.3) at the same endurance and simplifying gives:

$$S_a = \frac{S_{a_o}}{1 + \frac{S_m (S_{a_o} - S_e)}{(S_t - S_m)\ (S_t - S_e)}} \qquad (8.4)$$

Equation 8.4 may be compared with the previously mentioned Goodman and Gerber lines to examine the mean stress effect. Shown in Figure 8.4 together with the appropriate Goodman and Gerber lines is one of the interesting features of Jefferson's equation. Unlike the Goodman and Gerber equations, the alternating stress at a particular stress level does not vary in direct proportion to the alternating stress at zero mean in this equation. On an S-N diagram lines of constant mean stress, within a design range $\sigma_m < 0.3\,\sigma_t$, are widely dispersed at low fatigue life and very narrowly dispersed in the high cycle fatigue life. Consequently with a high alternating load, equation 8.4 shows a curve passing between the Goodman and Gerber lines, but with a low alternating load, that is at a higher level, the equation falls outside the lines (8.1-3). Also for low alternating stresses the Jefferson equation assumes that the fatigue life has little dependence upon the mean stress level for mean stresses up to about 60 percent of the ultimate tensile stress. This distinct characteristic differs the Jefferson equation from the others mentioned previously. With this property it has been found to fit the experimental data very well. The experimental evidence obtained from testing of a large number of bolts at the mean stresses up to 50% of the ultimate tensile strength has shown that the equation is in a close agreement with the very behaviour of the bolts under the mean loads of specified region.

9 METHOD OF ANALYSIS AND COMPUTATIONS

9.1 Introduction

During the last few decades the growing concern with fatigue has been the result of the greater demands made upon the reliability of components and structures. The most important area of the modern technology is the aircraft industry where the consequences of a failure may involve heavy loss of life. There is also pressure from designers because of the increasing competition of new materials and the need to avoid uneconomic over-design accruing from large safety factors. The fatigue strength of the material has already become the limiting factor in many branches of engineering. Many investigators have justified the use of statistical methods in fatigue by pointing to the differences found in the lives of nominally identical specimens when tested at the same stress level. Although it can be attributed to the various factors such as test techniques, specimen preparation, variations in the material and variability of the fatigue mechanism the causes of the scatter are not completely known. Scatter is usually greater in unnotched polished specimens than in notched or cracked specimens. The greater scatter especially at low stress levels in these smooth unnotched specimens can be attributed to the greater percentage of life needed to initiate small microcracks and then macrocracks. At higher stress levels a greater percentage of the fatigue life involves propagation of macrocracks. Tests involving only fatigue crack growth under constant amplitude conditions usually show scatter factors of 3 or less for identical tests (9.1.1). Thus it is reasonable to think that the greatest scatter in fatigue involves the initiation of microcracks and small macrocracks. In notched specimens and components, cracks form faster, and consequently a greater proportion of the total life involves crack propagation that has less scatter. By and large, the variability is such that the life at a particular stress can not be described by a single value and statistical distribution functions must be used. The problem of estimating unknown parameters in postulated mathematical relationships where there is scatter in the results, is essentially a statistical one. Also statistics is very helpful in economizing in the number of tests to be performed. Hence it can provide guidance on the design of experiments.

Conversely, it can be argued that numerous parts and components so far produced without giving due, if any, consideration to the statistical techniques have worked and are still working reasonably well.

So why are statistics an integral part of the design especially in fatigue? What has saved those products and consequently the people involved may be one of the three things: Either the products were designed with very large safety factors or the users operated them at very low loads; or the real criterion of these designs was not structural integrity but rigidity, wear, bearing area, etc.; or what is most likely, the Good Lord looked after them (9.1-2).

As stated above, if nominally identical specimens of the same material are tested at the same stress level, they will break after various numbers of cycles. The actual number of specimens tested represents only a small percentage of what is usually available. Though in practice an entire batch of a certain material would never be devoted solely to test specimens, the notation exists of a 'population' which consists of all possible specimens. If the complete population of specimens were tested to failure at a particular stress level, then a probability distribution would be defined by the number of specimens failing at different lives. Only then it would be possible to assign a probability to any particular specimen reaching a given life at this stress level. The sample of specimens actually tested provides an estimate of this probability distribution.

9.2 The log-normal distribution

Estimating the distribution of lives at a particular stress level, the so-called P/N relation at constant S, depends upon the assumed distribution of lives. When large numbers of tests have been carried out, the P/N distribution appears to be skew. It was suggested by Müller-Stock (9.2-1) that the logarithms of the lives can be considered as normally distributed. This has been shown as being a reasonable assumption in many cases (9.2-2). Freudenthal has shown that the log-normal distribution is a reasonable approximation to the distribution of the extent of progressive damage, if it is assumed that as a first approximation the effect of each cycle is directly proportional to the amount of damage produced (9.2-3).

Mathematically speaking, if x is a random variable which has a log-normal distribution, then x_1, logarithm of x will have a normal distribution as shown in figures 9.2a and 9.2b:

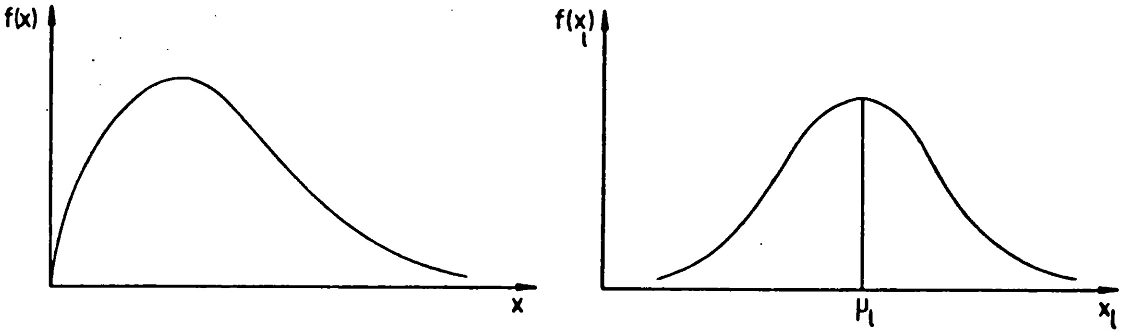

FIG.9.2a Log-normal Distribution

FIG.9.2b Normal Distribution

A random variable x has a log-normal distribution if

$$p = f(x_1) = \frac{1}{\sigma_1 \sqrt{2\pi}} \exp \left\{ \frac{-(x_1 - \mu_1)^2}{2\sigma_1^2} \right\} \quad 0 \leqslant x_1 \leqslant \infty \qquad (9.1)$$

where the following nomenclature is used:

$$x_1 = \log x \qquad (9.2)$$

$$\mu_1 = \int_{-\infty}^{+\infty} x_1 \, f(x_1) \, dx_1 = \text{log population mean} \ (9.3)$$

$$\sigma_1^2 = \int_{-\infty}^{+\infty} (x_1 - \mu_1)^2 \, f(x_1) \, dx_1 = \text{log population variance} \qquad (9.4)$$

$$E\,(\mu_1) = \bar{x}_1 = \frac{1}{n} \sum_{i=1}^{n} x_{1_i} = \text{log sample mean of n values} \qquad (9.5)$$

$$E(\sigma_1) = S_1 = \sqrt{\frac{\sum_{i=1}^{n} (x_{1_i} - \bar{x}_1)^2}{n - 1}}$$ = log sample standard deviation of n values (9.6)

$f(x_1)$ is the probability of occurrence of x_1. Rewritten in terms of fatigue parameters equation (9.1) becomes:

$$P = f(N_1) = \frac{1}{\sigma_1 \sqrt{2\pi}} \left[\frac{-(N_1 - \mu_1)^2}{2\sigma_1^2} \right] \quad 0 \leq N_1 \leq \infty \qquad (9.7)$$

where N_1 is the logarithm of life cycle, μ_1 is the mean of log N and σ^2 is the variance of log N. These two unknown parameters in this function are μ_1 and σ_1^2 and estimates of them may be found in the familiar manner:

$$N_1 = \log N \qquad (9.8)$$

$$\bar{N}_1 = E(\mu_1) = \frac{1}{n} \sum_{i=1}^{n} N_{1_i} \qquad (9.9)$$

$$S_1^2 = E(\sigma_1^2) = \frac{1}{n-1} \sum_{i=1}^{n} (N_{1_i} - \bar{N}_1)^2 \qquad (9.10)$$

The accuracy of these estimates depends upon the number of tests to be carried out.

Another distribution has been suggested by Weibull, known as the two parameter Weibull's distribution, and another the three parameter Weibull's distribution (9.2-4). Freudenthal and Gumbel produced a further distribution which has a theoretical foundation known as the weakest-link theory. In other words the theory states that 'no chain is stronger than its weakest link' (9.2-5). If the flaws or pre-existing cracks in a material are distributed at random, then different stresses will cause fracture at different points, and interest therefore focuses on the weakest points, that is the distribution of the smallest value. This theory was found to provide a good fit to data for copper and aluminium but it did not give good results with the material having a fatigue limit (9.2-6).

Both of these two distributions present more computational difficulties than the log-normal distribution.

9.3 The use of statistics

The statistical methods of calculation have been used throughout this work, to determine the following:

1) The average values of some mechanical and geometrical properties of the specimens.

2) To check the suitability of the assumption that the cycle lives of specimens at different stress levels follow a log-normal distribution.

3) To compute the parameters in the Jefferson's equation which fits the test data best and to this end to evaluate the estimates of the log-cycle lives.

4) To compute the confidence limits with a confidence coefficient $\gamma = 0.95$ for the mean of the log cycle lives of three or more specimens tested at each stress level.

5) To find the best fitting survival curves with 10%, 50% and 90% probability at a confidence level with $\gamma = 0.9$ for the 10 mm bolts.

The algorithm of each item above will be explained in detail.

9.3.1

In computing the average values of mechanical and geometrical properties of specimens, i.e. the mean and the standard deviation, equations (9.5) and (9.6) were used.

9.3.2

A comparison between the Weibull's two parameter distribution and the log-normal distribution was made at a stress level where the maximum number of 7 bolts were tested. The numerical computation can be seen in Appendix C. The results showed that there was no appreciable difference in the application of either distribution. Due to this and the fact that the computations involved in the log-normal distribution are more straightforward it was decided that this type of distribution would be used throughout in the analysis of test data.

The suitability of the application of the log-normal distribution for the cycle lives of specimens at the other different stress levels was then verified. The procedure used was as follows:

(a) The logarithms of life cycles to failure at each stress level were arranged in an increasing order.

(b) To each value of the log of life cycles a corresponding median rank was assigned according to the number of tests at that level using table (9.1) reproduced from reference (9.3-1).

(c) These data were plotted on normal probability paper with the log-life values on the abscissa and the median-rank values on the ordinate for each stress level.

(d) A best-fitting line was drawn through each set of points pertaining to the same stress level by the least-square technique and correlation coefficient for every line was calculated.

(e) Each correlation coefficient was tested for its significance by comparing it with the value given in table (9.2) reproduced from reference (9.3-2).

9.3.3

In order to find the best-fitting Jefferson's curve the following procedure was adopted:

(a) The mean and the standard deviation of the logarithms of cycle lives were calculated at each alt. stress level where at least three specimens were tested. Considering the increase in scatter of the lives of specimens as alt. stress is decreased at a constant mean stress, the number of specimens tested was increased with decreasing alt. stress. In so doing, the coefficients of variation in the log of cycle lives were kept under 3% throughout.

(b) As for the equation constant, B, various methods may be adopted to find its value for the best fitting S-N curve. One simple method is to draw the curve through a series of test points and then select one of them as appropriate boundary condition. If alt. stress and failure cycles associated with this point are substituted into the Jefferson's Equation, together with the mean and ultimate tensile stress and a value for endurance limit, then B can be solved by iteration.

Another method proposed by Dr Jefferson is to rearrange the equation into the following form:

$$\frac{1}{S_a} = \frac{1}{S_e} - \frac{1 - \dfrac{S_e}{S_t - S_m} . \log B}{S_e \log (N + B)} \qquad (9.11)$$

Plotting $\frac{1}{S_a}$ versus $\frac{1}{\log (N + B)}$ will give a straight line to the experimental data. The best fitting straight line, hence the best value of B, can then be obtained by making trial substitutions for B.

Although the endurance limit is required to apply either of the above methods, this limit is not available from the S-N curve. A new method developed by Dr Vogwell has been adopted in this study (9.3-3). Vogwell's method treats both the constant B and the endurance stress level as unknowns. Since two boundary conditions are necessary to find these unknowns two points are selected from among the test data. In this way the accuracy of the fit is improved because the curve will definitely pass at least through these two points. If Jefferson's Equation is rearranged to give the endurance limit:

$$S_e = \frac{S_a (S_t - S_m) \{\log (N + B) - \log B\}}{(S_t - S_m) + \log (N + B) - S_a \log B} \qquad (9.12)$$

Remembering that S is unique for the S-N curve, it is cancelled out by replacing the two test points into the equation as described in the Appendix D to obtain:

$$(S_{a1} - S_{a2})\ (S_t - S_m).\log (N_1 + B).\log (N_2 + B)$$

$$+ S_{a2}\ (S_t - S_m - S_{a1}).\log (N_1 + B).\log B$$

$$- S_{a1}\ (S_t - S_m - S_{a2}).\log (N_2 + B).\log B = 0 \qquad (9.13)$$

where (S_{a1}, N_1) and (S_{a2}, N_2) are the two test points mentioned above. After an initial value of B is assumed, Equation 9.13 can be solved by iterative method. In this work in order to improve the accuracy of Vogwell's method the mean of the cycle-lives of three tests in minimum was replaced into the equation instead of a single N value at each alternating stress level. In addition, the process of selecting two points for substitution was repeated for each available pair of points along the curve. A computer programme (Appendix A) was used to find values of B. This programme gives the corresponding S_e value for each B value calculated. Hence it has also been possible to compare the endurance limit at 10^7 cycles computed by Jefferson's Equation with the experimentally determined S_e value.

(c) The confidence limits for the mean of the log-cycle lives were calculated at each alt. stress level as described in 'A Guide for Fatigue Testing and the Statistical Analysis of Fatigue Data' (9.3-4). The procedure described is as follows:

A confidence level, γ, is selected, bearing in mind that there is a risk of $(1 - \gamma)$ that the interval to be constructed will not contain the mean. Also the greater the confidence limit the wider the interval.

After choosing γ, $\beta_1 = (1 - \gamma)/2$ and $\beta_2 = (1 + \gamma)/2$ are computed. Corresponding t_{β_1} and t_{β_2} values are read from table (9.3) 'reproduced from the above reference where they are tabulated as functions of (n - 1), - the degrees of freedom then the confidence limits for the mean are given by the following formulae:

$$\overline{N}_\ell + t_{\beta_1}.\frac{s}{\sqrt{n}} \qquad (9.14)$$

$$\overline{N}_{\ell} + t_{\beta_2}\cdot\frac{s}{\sqrt{n}} \qquad (9.15)$$

where

$\overline{N}_{\ell}$ is the mean of the log-cycle lives at a particular level

s is the standard deviation of the log-cycle lives at that level

n is the number of tests done at the same level.

(d) The P-S-N curves of the 10 mm dia-bolts were plotted for 10%, 50% and 90% probability as shown in Figures (9.3a), (9.3b), (9.3c) at three different mean load levels. Consideration given to the following argument led to calculate and draw these curves through those stress levels where at least three tests were done. It has been accepted that the logarithm of cycle-lives of specimens used in this work is a normally distributed random variable. If the parameters of this distribution, μ_{ℓ} and σ_{ℓ}, were known for the whole population of bolts from which the sample used in this work was taken, then for any preassigned percentage P, a number K, could be determined so that P percent of the population would have cycle lives exceeding $(\mu_{\ell} - K\sigma_{\ell})$. However, μ_{ℓ} and σ_{ℓ} were unknown and could only be estimated by information obtained from the sample drawn from population. In this case, it can only be determined a number, k, such that the probability of random variable $(\overline{N}_{\ell} - ks)$ not exceeding $(\mu_{\ell} - K\sigma_{\ell})$ is exactly γ, where γ is a confidence level chosen in advance, $\overline{N}_{\ell}$ and s are the mean and the standard deviation of the cycle lives of n specimens respectively. Hence one can say that with a confidence level of γ, that at least P percent of the population is greater than $\overline{N}_{\ell}$ - ks. The numbers k, one-sided tolerance factors, are functions of P, γ and n.

If the problem of constructing a specific S-N curve of the bolts say, 75 percent survival is considered any point $(S_1 N_1)$ on the curve might be expected to give the following information:

If the alternating stress is S_1, then 75 percent of the bolts to be tested will survive N_1 cycles. Since the parameters of the fatigue life distribution are not known, the above defined curve cannot be constructed. Instead, one can construct a curve whereon any point (S_2, N_2) has the following meaning: If the alt. stress is S_2, then with a confidence level of γ, at least 75 percent of the specimens to be tested will survive N_2 cycles.

The 10%, 50% and 90% probability curves with a confidence level of $\gamma = 0.90$ were plotted at three mean load levels. To this end, k values were taken from the table (9.4) in the ref (9.3-4) by considering the percent survival, P, the confidence level γ and the sample size, n. The value ($\overline{N}_{\ell}$ - ks) was then the appropriate abscissa for that particular alt. stress level on the curve. The equations of the best fitting curves are listed in table (9.5).

9.4 Computer Programs

Two different computer programs were used in this work in addition to those used for routine statistics. By means of the first one the parameters B and S_e in the Jefferson's Equation were calculated for each pair of points from the data belonging to a certain mean load level. Most of the points have the mean of a group of tests as their abscissa at each stress level.

A copy of this program is given in Appendix A. In order to plot the best fitting Jefferson's Curve, the second program was used. This program in multics language plots the same curve on two different sheets. In one it superimposes all the test points with the confidence limits, in the other it locates the means of the groups together with isolated test points. A copy of this second program is given in Appendix B.

10. DISCUSSION

10.1 Introduction

A survey of the history of fatigue research shows how the occurrence of fatigue failures in service conditions compelled a few engineers to start designing elementary testing machines and making fatigue tests. As use of higher strength steels increased and strong aluminium alloys were introduced the demand for fatigue data increased. The literature survey shows that with the development of the fatigue testing machines and their use in a large number of laboratories the fatigue research progresses on an ever-increasing scale. However, the majority of this work is usually devoted to a particular engineering product with the 'ad hoc' objective of explaining the failure after the event.

From 1950 onwards the amount and diversity of published reports giving fatigue data exceed the ability of engineers to comprehend and use the data. Most of the information was produced for an 'ad hoc' purpose and in an attempt to use it for another 'ad hoc' purpose the engineer finds himself supplied with some results obtained for a component that has no stress concentration or welding which may be in contrast to his case under study. Even with a pertinent report available he usually comes across the difficulty of having a missing parameter. The missing one which may be the mean stress or actual alloy strength is due to the large number of variables involved or the 'ad hoc' nature of the research.

This research has attempted to analyze one consistent set of fatigue tests to determine a theoretical method of presenting the results. In common with all theoretical methods of analysing experimental data an empirical equation is used. If the fact that even Hooke's Law is an empirical equation is recalled, the important feature of the equation used appears to be the minimum number of constants involved. These constants are few and are meaningful properties of the material, particularly the tensile strength and the endurance stress.

The fatigue tests have been made on both nuts/bolts and screwed bar in tension and good agreement has been obtained over a wide range of variables in a large number of individual tests. The reduction of the data has been programmed for a computer such that the fatigue curves can be plotted by the computer either giving mean life data or data at a chosen statistical confidence level.

Tests on the bolts and screwed bar in two different test machines have been analyzed at each mean stress and it is considered that the accuracy is sufficient to show meaningful differences between variables and these will be discussed below. The constants are recorded in Table 10.1 and only three are required. The tensile strength is obtained by a static tensile test and is a basic data item, further the normal statistical scatter is meaningful and can be incorporated into the analysis of the fatigue data. The equation constant B is shown in Table 10.1 to have a range of values but as it is used in logarithmic form this has a very small effect in the case of bolts. Taking into consideration the characteristic $S_a - S_m$ curve of the bolts it could be reasonable to state one value which could be used for a range of bolts, i.e. a value of B for the particular application. The third constant is the endurance stress and it is shown that its value can be defined with reasonable confidence for the bolts and screwed bar.

If the wide range of fatigue research which is in progress in many laboratories could be recorded as constant B and endurance stress values of known tensile strength, it is considered that progress would be made in providing useful data for the design engineer.

10.2 The scatter in screwed bar fatigue data

The first series of fatigue tests were made on screwed bars from various sources and therefore of varying strength. As figure 5.3 shows quite a wide scatter band exists to cover this variation. The use of two machines and variation of applied mean stress are reflected in this figure. The results are typical of some of the published data. The graph also shows the boundaries of two standard deviations on the static properties and these include the great majority of results. The scatter in these results is therefore not due to inherent scatter in fatigue testing of screwed bar but is largely explainable by controllable basic variables. The effect of using two testing machines is clearly shown in Figures 10.5 and 10.6.

In the Figures 5.4 and 5.5 the mean loads have been separated and the scatter is seen to reduce. In fact the few results which are near the 2s lines are explainable by their tensile strength properties. Consideration of the different groups of data on the graphs shows that the scatter is low within the tests made within controlled variables, that is, one material, one testing machine and one mean load.

There is a definite trend in the fatigue life as related to the tensile strength of the bar. Through the lower and middle range of life the higher strength material gives better results as would be expected. This difference does tend to disappear as the stress levels approach the fatigue limit.

An additional variable, not mentioned above, is the method of attaching the screwed bar to the testing machine. As described in Section 5.1 two different methods were used and these lead to a difference in fatigue lives which is fully explained by the nature of the local stresses at the joint.

10.3 The statistical scatter in screwed bar tests

The results shown in Figures 5.6 onwards were obtained from a series of tests which were planned to produce statistical data. These results show that fatigue tests with controlled variables do produce good results. The specimens were commercial quality mild steel screwed bar and their production involves the minimum of manufacturing control. It is therefore likely that much of the scatter shown is actually variation in the screwed bar.

Where the test results are plotted as a single point mean value at each stress level as seen in Figure 5.7 and others the results are very good. This would appear to be an optimum method of obtaining mean fatigue life data. The results also fit the theoretical equation given with each graph and give considerable support to its accuracy. Typical mean life results are given in Figures 10.1 and 10.3 but it would be wiser to provide design data at a 95%

confidence level as shown in Figures 10.2 and 10.4.

By comparing the values of the constants in the equations given alongside the graphs, that is, B and S_e, it is possible to see that there exist definite trends between these values. Usually the variation in the constants is small and they are related to each other. Further work can usefully be done on the trends in these constants. Of particular note in these trends is that Figure 5.14 which shows that the endurance stress varies with the mean stress according to a Gerber relation.

10.4 The fatigue properties of bolts

The main interest in this research is in tension bolts and the screwed bar tests were introduced to help in this understanding. In principle the bolts follow the same pattern as for screwed bar but the tests were all planned to determine statistical patterns in the fatigue curves. The results are shown in the figures of Chapter 6, and again a good agreement is obtained with the theoretical equation. The mean values at each stress level gives good agreement with the theoretical curve as shown particularly in Figure 6.7.

There are however some fundamental differences from the screwed bar. It is not surprising that the fatigue life of the bolts is found to be shorter than that of the direct loaded screwed bar due to the greater local stress path at the head and the nut. Of considerable interest is the fact that the endurance stress for bolts is largely independent of the mean load as shown in Figure 6.9. This result has been noted by other research workers and this leads to one value of the endurance stress in the fatigue equation. As there appears to be a relation between the constants B and S_e it follows that the value of B is constant for the bolts over the practical range of mean load.

The results of Table 10.1 show that for the 10 mm bolts the values of B and S_e show small variations. The mean values of the constants were used to extrapolate to higher levels of mean load for which test results are available in Chapter 6. The correlation between the extrapolation and the test results at 10^7 and 10^8 cycles was found to be very good, that is well within the expected scatter band. This result brings about the possibility that the fatigue curve for bolts can be defined by one value for B and one for S_e for design purposes.

The presentation of typical results both at mean load and at 95% confidence level are shown in Figures 10.7 to 10.10. This presentation does give clear comparisons as is demonstrated by Figure 10.10 where the high tensile strength of the 12 mm bolts in the Avery machine affects the shorter lives quite clearly. At longer lives the differences between the two bolts and the two testing machines largely balance each other.

10.5 Basic fatigue relationships

In chapter 7 the fatigue equations which have been used for many years are reviewed. These equations were originally designed to relate the endurance stress to mean but have been extended to shorter fatigue lives. At the endurance limit they do fit the results quite well although each one suits a particular metal. Some of the equations have the disadvantage of requiring too many empirical constants thus many fatigue tests and laborious computational methods to establish the equation.

The equation proposed by Dr Jefferson is discussed in Chapter 8, and this has been shown to fit the experimental results. In this form it has a constant endurance stress irrespective of mean stress and this suits the bolt results reliably well. A comparison of the equation with the Gerber and Goodman lines is given in Figure 8.4 and it is seen that the equations diverge from each other at lives towards the endurance limit. For bolts the basic Jefferson equation is most suitable and for other applications it is possible to incorporate the Goodman and Gerber equations into the Jefferson equation.

The effect of varying the mean stress, the constant B and the endurance stress is shown in Figures 8.1 to 8.3 and these show how the equation is adaptable to the type of component which is considered.

11 CONCLUSIONS

A historical survey of fatigue research since the time of Wöhler has shown an ever-increasing importance of fatigue as a primary cause of failure. This situation has created an increasing quantity of 'ad hoc' test data. This research has demonstrated a method of presenting comprehensive fatigue data (for bolts as a typical example) in a compact manner which is useful to engineers.

A large number of tests were performed on both bolts and screwed bar and the results have been analyzed by a statistical technique which has been programmed such that a computer will plot fatigue design curves at any chosen confidence level.

A fatigue equation developed at the University of Bath was applied to the presentation of the data obtained from screwed bar and bolt tests. The accuracy of the equation was increased by introducing computer programs to determine the equation constants in the equation and to plot the data.

The tests were planned to carefully control all the relevant variables and it was particularly found that the mean of up to 7 tests at each stress level gave a good correlation with the equation. The curve would lie within the 95% confidence limits produced by the computer for the mean of the groups at each stress level.

The fatigue limits of screwed bars at 10^7 cycles followed the pattern of Gerber parabola showing a dependence on the mean stress level. In contrast to this the nuts and bolts exhibited almost no dependence on the mean stress over a practical design range.

The bolts showed shorter fatigue life than the screwed bar due to the higher stress concentration by the load transfer through a nut and under the head.

Specimens tested in the Vibraphore showed noticeably longer fatigue life than those tested in the Avery.

Specimens of high tensile strength displayed longer fatigue lives over the low and middle life range but the difference tends to disappear as they approach to 10^6 cycles.

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TABLES AND FIGURES

TABLE 3.1

PROPERTIES OF BOLTS AND NUTS

	Unit	M-12 Bolt	M-10 Bolt
Designation	-	CRF/M88 Hexagon Black Bolt	GKN/ISOM Hexagon Black Bolt
Material	-	Steel	Steel
Method of Production	-	-	Cold rolled
Tensile Strength	N/mm^2	981*	590*
.1% Proof Stress	N/mm^2	935	513
Hardness	VHN	320	175
Length	mm	100	93
Length of Shank	mm	70	63
Length of Thread	mm	30	30
Pitch (course)	mm	1.75	1.5
Nominal Shank Dia.	mm	12	10
Measured Shank Dia.	mm	11.80	9.75
Shank Area (calc.)	mm^2	109.4	74.7
Nominal Minor Dia.	mm	10.11	8.16
Nominal Pitch Dia.	mm	10.86	9.03
Calc. Pitch Dia.	mm	10.66	8.78
Nom. Tensile Stress Area	mm^2	84.3	58
Calc. Tensile Stress Area	mm^2	81.8	55.3
Height of Head	mm	8	7
Radius of Thread Root	mm	0.6	0.4
Width Across Flats	mm	18.5	16.6
Width Across Corners	mm	21.4	19

	Unit	M-12 Nut	M-10 Nut
Designation	-	V8 Black Nut	FCW8 Black Nut
Material	-	Steel	Steel
Measured Hardness	VHN	240	208
Thickness	mm	10	7.6
Pitch (course)	mm	1.75	1.5
Width Across Flats	mm	18.50	16.60
Width Across Corners	mm	21.4	19

* Based on the nominal tensile stress area

TABLE 3.2

PROPERTIES OF SCREWED BAR

	Unit	Screwed Bar 1st Batch	Screwed Bar 2nd Batch	Screwed Bar 3rd Batch
Designation	-	150 M12x120	150 M12x120	150 M12x120
Material	-	Steel	Steel	Steel
Method of Production	-	Cold rolled	Cold rolled	Cold rolled
Tensile Strength	N/mm^2	672*	538*	512*
.1% Proof Stress	N/mm^2	520	420	400
Hardness	VHN	208	166	152
Elastic Modulus	kN/mm^2	205	205	205
Length	mm	120	120	120
Nominal Dia.	mm	12	12	12
Measured Dia.	mm	11.84	11.84	11.84
Pitch (course)	mm	1.75	1.75	1.75
Nom. Minor Dia.	mm	9.85	9.85	9.85
Measured Minor Dia.	mm	9.75	9.75	9.75
Nom. Pitch Dia.	mm	10.86	10.86	10.86
Calc. Pitch Dia.	mm	10.70	10.70	10.70
Nom. Tensile Stress Area	mm^2	84.3	84.3	84.3
Calc. Tensile Stress Area	mm^2	83.7	83.7	83.7
Calculated Core Area	mm^2	74.7	74.7	74.7

* Based on the calculated core area

Notes:

1. Dimensions measured and calculated are within the BS 4190 limits.
2. Hardness and tensile strength of the specimens were calculated by statistical averaging.
3. Nominal tensile stress area used for bolts is given by the following formula:

$$\text{NOM. TENSILE STRESS AREA} = \frac{\pi}{4}\left(\frac{d_{minor} + d_{pitch}}{2}\right)^2$$

Nominal tensile stress area was used to abide by the regulations set by BS 4190.

TABLE 3.3

STATISTICS OF MECHANICAL PROPERTIES OF SPECIMENS

Specimen Type	Breaking Load (kN)	Tensile Strength (N/mm^2)		Yield Strength (N/mm^2)	No. of Tests	Hardness (V.H.N)		No. of Tests
		Mean	Stan. Dev.			Mean	Stan. Dev.	
ϕ12-Screwed Bar 1st Batch	50.2	672	-	520	1	208	-	1
ϕ12-Screwed Bar 2nd Batch	40.2	538	21.46	420	12	166	4.74	5
ϕ12-Screwed Bar 3rd Batch	38.2	512	13.12	400	12	152	4.62	5
ϕ12-Bolt	81.7	981	21	935	8	320	4.18	5
ϕ10-Bolt	34.2	590	15	513	8	175	5.02	5
ϕ12-Nut	-	-				240	-	1
ϕ10-Nut	-	-				208	-	1

TABLE 5.1

DATA OF EARLY TESTS OF ϕ12-SCREWED BAR

A - AVERY-SCHENCK TESTS

	Tensile Strength N/mm^2	Mean Load kN	Mean Stress N/mm^2	Alternating Load kN	Alternating Stress N/mm^2	Failure Cycles (N)
d	672	10	134	22	295	5,100
d	672	10	134	21	281	29,300
d	672	10	134	17.5	234	85,000
d	672	10	134	12.5	167	642,000
tn	538	10	134	24	321	5,800
tn	538	10	134	20	268	28,500
tn	538	10	134	15	201	72,000
tn	538	10	134	10	134	532,000
tn	538	10	134	7.5	101	1,830,000
tn	538	10	134	6.5	87	5,400,000*
d	538	20	268	16	214	29,000
d	538	20	268	15	201	35,000
d	538	20	268	12.5	167	49,700
d	538	20	268	11	147	70,000
d	538	20	268	10	134	122,600
d	538	20	268	8.5	114	209,800
d	538	20	268	7	94	383,300
d	538	20	268	6	81	1,011,550
d	538	20	268	5	67	2,213,900
d	538	20	268	4	54	14,200,000*

* unbroken specimen

tn through nuts tests

d direct tests

TABLE 5.2a

DATA OF EARLY TESTS OF ϕ12-SCREWED BAR

B - VIBRAPHORE TESTS

	Tensile Strength N/mm^2	Mean		Alternating		Failure Cycles N	
		Load kN	Stress N/mm^2	Load kN	Stress N/mm^2	N_1 1st test	N_2 2nd test
d	538	10	134	20	268	25,000	-
d	538	10	134	18	241	60,200	-
d	538	10	134	16	214	125,000	-
d	538	10	134	15	201	234,900	614,900
d	538	10	134	14	188	181,100	-
d	538	10	134	13	174	545,700	-
d	538	10	134	12.5	167	426,000	343,000
d	538	10	134	10.5	141	4,573,000	-
d	538	10	134	10	134	1,874,000	5,595,000
d	538	10	134	9	121	4,453,000	6,002,000
d	538	10	134	8	107	14,000,000*	54,000,000*
tn	512	20	268	18	241	34,700	-
tn	512	20	268	17	228	83,000	-
tn	512	20	268	15	201	188,000	-
tn	512	20	268	14	188	234,500	-
tn	512	20	268	12	161	482,500	-
tn	512	20	268	10	134	882,200	-
tn	512	20	268	8	107	1,632,800	-
tn	512	20	268	6	81	2,760,400	-
tn	512	20	268	5	67	14,770,000*	-
tn	512	20	268	4	54	11,000,000*	-

* unbroken specimen

tn through nuts tests

d direct tests

TABLE 5.2b

DATA OF EARLY TESTS OF ϕ12-SCREWED BAR

B - VIBRAPHORE TESTS (continued)

	Tensile Strength N/mm^2	Mean		Alternating		Failure Cycles N	
		Load kN	Stress N/mm^2	Load kN	Stress N/mm^2	N_1 1st test	N_2 2nd test
d	538	20	268	15	201	46,000	57,000
d	538	20	268	14	188	74,000	-
d	538	20	268	13	174	106,300	-
d	538	20	268	12	161	167,000	-
d	538	20	268	11	147	265,000	-
d	538	20	268	10	134	265,700	310,000
d	538	20	268	9	121	829,000	-
d	538	20	268	8.5	114	2,333,600	-
d	538	20	268	8	107	1,834,000	-
d	538	20	268	7	94	13,800,000*	5,948,000
d	538	20	268	6	81	12,040,000*	-
d	538	20	268	5	67	14,050,000*	-
tn	538	28	375	9	121	251,500	-
tn	538	28	375	8	107	341,800	-
tn	538	28	375	7	94	40,000,000*	-

* unbroken specimen

tn through nuts tests

d direct tests

TABLE 5.3

DATA OF THE ϕ12-SCREWED BAR AVERY-SCHENCK TESTS

	MEAN LOAD: 10 kN MEAN STRESS: 134 N/mm²									
	ALT. LOAD: 24 kN		ALT. LOAD: 20 kN		ALT. LOAD: 15 kN		ALT. LOAD: 10 kN		ALT. LOAD: 7.5 kN	
	S_a = 321 N/mm²		S_a = 268 N/mm²		S_a = 201 N/mm²		S_a = 134 N/mm²		S_a = 100 N/mm²	
	N	N_ℓ	N	N_ℓ	N	N_ℓ	N	N_ℓ	N	N_ℓ
1	5,800	3.763	25,900	4.413	64,400	4.809	352,000	5.547	1,830,000	6.262
2	6,500	3.813	28,500	4.455	72,000	4.857	410,300	5.613	2,319,000	6.365
3	7,200	3.857	34,100	4.533	90,200	4.955	532,000	5.726	3,242,000	6.511
S_ℓ	0.047		0.061		0.074		0.091		0.125	
$\bar{N}_\ell$	3.811		4.467		4.874		5.629		6.379	
$S_\ell/\bar{N}_\ell$	1.23%		1.36%		1.52%		1.61%		1.96%	
$\bar{N}$	6,471		29,309		74,760		425,272		2,395,153	
N_{lower}	4,947		20,680		49,010		252,978		1,171,316	
N_{upper}	8,466		41,537		114,212		716,006		4,890,190	

	MEAN LOAD: 20 kN MEAN STRESS: 268 N/mm²							
	ALT. LOAD: 16 kN		ALT. LOAD: 12.5 kN		ALT. LOAD: 8.5 kN		ALT. LOAD: 6 kN	
	S_a = 214 N/mm²		S_a = 167 N/mm²		S_a = 114 N/mm²		S_a = 80 N/mm²	
	N	N_ℓ	N	N_ℓ	N	N_ℓ	N	N_ℓ
1	26,700	4.427	49,700	4.696	179,800	5.255	785,300	5.895
2	29,000	4.462	54,300	4.735	209,800	5.322	1,011,500	6.005
3	32,300	4.509	64,500	4.810	245,600	5.390	1,772,000	6.248
S_ℓ	0.041		0.058		0.068		0.181	
$\bar{N}_\ell$	4.466		4.747		5.322		6.049	
$S_\ell/\bar{N}_\ell$	0.92%		1.22%		1.27%		2.99%	
$\bar{N}$	29,242		55,847		210,055		1,120,297	
N_{lower}	23,132		40,087		142,293		397,785	
N_{upper}	36,965		77,802		309,612		3,150,299	

TABLE 5.4

FATIGUE TEST DATA OF THE ϕ12-SCREWED BAR TESTED ON THE VIBRAPHORE

	MEAN LOAD: 10 kN MEAN STRESS: 134 N/mm^2								MEAN LOAD: 20 kN MEAN STRESS: 268 N/mm^2							
	ALT. LOAD: 20 kN		ALT. LOAD: 16 kN		ALT. LOAD 12.5 kN		ALT. LOAD: 10 kN		ALT. LOAD: 15 kN		ALT. LOAD: 12 kN		ALT. LOAD: 10 kN		ALT: LOAD: 8 kN	
	S_a = 268 N/mm^2		S_a = 214 N/mm^2		S_a = 167 N/mm^2		S_a = 134 N/mm^2		S_a = 201 N/mm^2		S_a = 161 N/mm^2		S_a = 134 N/mm^2		S_a = 107 N/mm^2	
	N	N_ℓ	N	N_ℓ	N	N_ℓ	N	N_ℓ	N	N_ℓ	N	N_ℓ	N	N_ℓ	N	N_ℓ
1	25,000	4.405	90,500	4.957	343,000	5.535	1,874,000	6.273	46,000	4.663	143,400	5.157	266,000	5.425	1,401,000	6.146
2	30,400	4.483	108,700	5.036	426,000	5.629	3,734,200	6.572	52,800	4,723	167,000	5.223	310,000	5.491	1,834,000	6.263
3	33,800	4.529	125,000	5.097	503,700	5.702	5,950,000	6.775	57,000	4.756	205,000	5,312	418,000	5.621	2,507,000	6.399
S_ℓ	0.063		0.070		0.084		0.253		0.047		0.078		0.1		0.127	
$\bar{N}_\ell$	4.472		5.03		5.622		6.54		4.714		5.231		5.512		6.269	
$S_\ell/\bar{N}_\ell$	1.40%		1.40%		1.49%		3.87%		1.0%		1.49%		1.81%		2.02%	
$\bar{N}$	29,648		107,152		418,794		3,467,369		51,761		170,085		325,337		1,859,231	
N_{lower}	20,682		71,815		259,096		816,396		39,566		108,982		183,544		898,895	
N_{upper}	42,501		159,876		676,922		14,726,494		67,715		265,854		575,785		3,839,644	

TABLE 5.5

DATA OF ISOLATED TEST POINTS

ϕ12-SCREWED BAR - AVERY SCHENCK TESTS				
Test No.	Mean Load (kN)	Alt. Load (kN)	Alt. Stress (N/mm^2)	Failure Cycles, N
1	20	15	201	35,000
2	20	11	147	70,000
3	20	10	134	122,600
4	20	7	93	383,300
5	20	5	67	2,213,900
ϕ12-SCREWED BAR - VIBRAPHORE TESTS				
1	10	18	241	60,200
2	10	15	201	234,900
3	10	14	188	181,100
4	10	13	174	545,700
5	10	12	167	782,200
6	10	9	121	6,002,000
1	20	18	241	34,700
2	20	14	188	74,000
3	20	11	147	265,000
4	20	9	121	829,000

TABLE 5.6

FATIGUE LIMIT DATA OF THE 12 mm SCREWED BAR

Mean		Alternating		Life Cycles	
Load (kN)	Stress (N/mm²)	Load (kN)	Stress (N/mm²)	N	
0	0	10	134	5,172,000	broken
0	0	8	107	10^7	unbroken
0	0	9	<u>121</u>	10^7	unbroken
0	0	9.5	127.5	6,349,000	broken
5	67	9	121	3,918,400	broken
5	67	8	107	10^7	unbroken
5	67	8.5	<u>113.5</u>	10^7	unbroken
10	134	8	107	10^7	unbroken
10	134	8	<u>107</u>	10^7	unbroken
15	201	8	107	2,390,000	broken
15	201	7.5	<u>100</u>	10^7	unbroken
20	268	7	94	10^7	unbroken
20	268	7	94	5,948,000	broken
20	268	6.5	<u>87</u>	10^7	unbroken
25	335	6	81	2,924,000	broken
25	335	5	67	10^7	unbroken
25	335	5.5	<u>73.5</u>	10^7	unbroken
30	402	5	67	3,172,000	broken
30	402	4	<u>54</u>	10^7	unbroken
30	402	4.5	60.5	6,312,500	broken
35	469	3	41	2,118,000	broken
35	469	2.5	<u>35</u>	10^7	unbroken

TABLE 6.1

FATIGUE LIMIT DATA OF THE 10 mm BOLT

Mean		Alternating		Life Cycles	
Load (kN)	Stress (N/mm^2)	Load (kN)	Stress (N/mm^2)	N	
7.5	129	3	51.7	2,917,000	broken
7.5	129	2.5	43.1	10^7	unbroken
7.5	129	2.75	47.4	7,009,000	broken
7.5	129	2.6	<u>44.8</u>	10^7	unbroken
10	172	3	51.7	7,586,000	broken
10	172	2.5	43.1	10^7	unbroken
10	172	2.6	44.8	10^7	unbroken
10	172	2.75	<u>47.4</u>	10^7	unbroken
15	259	3	51.7	4,127,000	broken
15	259	2.75	47.4	10^7	unbroken
15	259	2.85	<u>49.1</u>	10^7	unbroken
15	259	2.9	50	9,550,600	broken
20	345	3.2	55.2	2,590,000	broken
20	345	3.1	53.4	7,657,000	broken
20	345	3	<u>51.7</u>	10^7	unbroken
20	345	2.85	49.1	10^7	unbroken
25	431	3	51.7	3,305,000	broken
25	431	2.5	43.1	10^7	unbroken
25	431	2.75	<u>47.4</u>	10^7	unbroken
25	431	2.85	49.1	8,127,000	broken
30	517	2.5	43.1	1,700,000	broken
30	517	2.25	38.8	4,935,000	broken
30	517	2.15	<u>37.1</u>	10^7	unbroken

TABLE 6.2

DATA OF ϕ10 BOLT AT 10 kN MEAN LOAD VIBRAPHORE TESTS

	ALT. LOAD: 10 kN		ALT. LOAD: 9 kN		ALT. LOAD: 8 kN		ALT. LOAD: 7 kN		ALT. LOAD: 6 kN		ALT. LOAD: 5 kN		ALT. LOAD: 4 kN	
	S_a = 172 N/mm^2		S_a = 155 N/mm^2		S_a = 138 N/mm^2		S_a = 121 N/mm^2		S_a = 103 N/mm^2		S_a = 86 N/mm^2		S_a = 69 N/mm^2	
	N	N_ℓ	N	N_ℓ	N	N_ℓ	N	N_ℓ	N	N_ℓ	N	N_ℓ	N	N_ℓ
1	47,300	4.675	76,600	4.884	127,300	5.105	240,000	5.380	385,000	5.585	626,000	5.747	1,248,100	6.096
2	55,200	4,742	81,000	4,908	178,200	5.251	289,000	5.461	510,000	5.708	642,000	5.808	1,456,000	6.163
3	62,900	4.799	109,400	5.039	194,000	5.288	369,000	5.567	577,000	5.761	912,000	5.960	2,040,000	6.310
4							447,000	5.650	614,000	5.788	958,700	5.982	2,553,500	6.407
5											1,280,000	6,107	2,942,000	6.469
6													3,782,000	6.578
7													4,212,000	6.624
S_ℓ	0.062		0.083		0.097		0.118		0.090		0.130		0.200	
$\overline{N}_\ell$	4.739		4.944		5.215		5.515		5.711		5.931		6.378	
$S_\ell/\overline{N}_\ell$	1.31%		1.69%		1.86%		2.15%		1.58%		2.19%		3.136%	
$\overline{N}$	54,786		87,835		163,933		326,964		513,452		852,707		2,388,597	
N_{lower}	38,466		54,695		94,230		212,510		369,743		588,000		1,558,825	
N_{upper}	78,149		141,272		285,636		504,220		714,661		1,237,722		3,657,653	

TABLE 6.3

DATA OF ϕ10 BOLT AT 15 kN MEAN LOAD VIBRAPHORE TESTS

	ALT. LOAD: 10 kN		ALT. LOAD: 9 kN		ALT. LOAD: 8 kN		ALT. LOAD: 7 kN		ALT. LOAD: 6 kN		ALT. LOAD: 5 kN		ALT. LOAD: 4 kN	
	S_a = 172 N/mm^2		S_a = 155 N/mm^2		S_a = 138 N/mm^2		S_a = 121 N/mm^2		S_a = 103 N/mm^2		S_a = 86 N/mm^2		S_a = 69 N/mm^2	
	N	N_ℓ	N	N_ℓ	N	N_ℓ	N	N_ℓ	N	N_ℓ	N	N_ℓ	N	N_ℓ
1	27,900	4.446	39,200	4.593	62,500	4.796	95,600	4.980	229,300	5.360	429,100	5.633	859,900	5.934
2	32,100	4.507	45,500	4.658	66,700	4.824	104,300	5.018	268,000	5.428	454,000	5.657	898,800	5.954
3	35,700	4.553	51,000	4.708	82,100	4.914	124,700	5.096	278,000	5.444	477,400	5.679	972,300	5.988
4							130,000	5.114	394,000	5.595	661,600	5.821	989,400	5.995
5											720,000	5.857	1,187,600	6.075
6													1,558,700	6.193
7													1,613,000	6.208
S_ℓ	0.054		0.058		0.062		0.064		0.099		0.102		0.112	
$\bar{N}_\ell$	4.502		4.653		4.845		5.052		5.457		5.729		6.050	
$S_\ell/\bar{N}_\ell$	1.22%		1.24%		1.28%		1.26%		1.81%		1.78%		1.86%	
$\bar{N}$	31,769		44,978		69,930		112,720		286,253		536,290		1,120,912	
N lower	28,381		32,286		49,100		89,128		199,179		400,118		883,662	
N upper	43,257		62,660		99,752		142,556		411,869		717,483		1,424,667	

TABLE 6.4

DATA OF ϕ10 BOLT AT 20 kN MEAN LOAD VIBRAPHORE TESTS

	ALT. LOAD: 10 kN		ALT. LOAD: 9 kN		ALT. LOAD: 8 kN		ALT. LOAD: 7 kN		ALT. LOAD: 6 kN		ALT. LOAD: 5 kN		ALT. LOAD: 4 kN	
	S_a = 172 N/mm^2		S_a = 155 N/mm^2		S_a = 138 N/mm^2		S_a = 121 N/mm^2		S_a = 103 N/mm^2		S_a = 86 N/mm^2		S_a = 69 N/mm^2	
	N	N_ℓ	N	N_ℓ	N	N_ℓ	N	N_ℓ	N	N_ℓ	N	N_ℓ	N	N_ℓ
1	17,400	4.241	24,800	4.394	45,800	4.461	74,000	4.869	142,300	5.153	30.20×10^4	5.480	5.3×10^5	5.724
2	18,500	4.267	27,200	4.435	52,700	4.722	80,800	4.907	156,600	5.195	33.20×10^4	5.521	6.492×10^5	5.812
3	20,100	4.303	29,400	4.468	54,900	4.740	85,000	4.929	180,000	5.255	35.00×10^4	5.544	7.430×10^5	5.871
4							100,300	5.001	204,500	5.311	41.40×10^4	5.617	7.978×10^5	5.902
5											47.60×10^4	5.677	8.496×10^5	5.929
6													10.293×10^5	6.015
7													10.538×10^5	6.023
S_ℓ	0.031		0.037		0.0414		0.056		0.069		0.079		0.107	
$\bar{N}_\ell$	4.270		4.432		4.708		4.927		5.229		5.568		5.896	
$S_\ell/\bar{N}_\ell$	.73%		.83%		.88%		1.13%		1.32%		1.41%		1.81%	
$\bar{N}$	18,621		27,068		51,011		84,431		199,388		369,828		787,590	
N lower	15,597		21,885		40,292		68,859		131,610		294,973		626,494	
N upper	22,231		33,408		64,681		103,762		226,820		463,679		988,743	

TABLE 6.5

DATA OF ISOLATED TEST POINTS

ϕ10-BOLT - VIBRAPHORE TESTS				
Test No.	Mean Load kN	Alt. Load kN	Alt. Stress N/mm^2	Failure Cycles, N
1	15	15	259	7,000
2	15	15	259	7,200
3	15	14	241	9,900
4	15	14	241	10,500
5	15	13	224	9,500
6	15	13	224	9,900
7	15	12	207	14,400
8	15	12	207	15,200
9	15	11	190	19,400
10	15	11	190	20,000
1	20	13	224	8,700
2	20	13	224	9,100
3	20	12	207	9,400
4	20	12	207	10,000
5	20	11	190	13,400
6	20	11	190	14,200

TABLE 6.6

DATA OF THE ø12-BOLT AT 20 kN MEAN LOAD (AVERY-SCHENCK TESTS)

	ALT. LOAD: 17 kN		ALT. LOAD: 16 kN		ALT. LOAD: 15 kN		ALT. LOAD: 14 kN		ALT. LOAD: 13 kN		ALT. LOAD: 12 kN		ALT. LOAD: 11 kN	
	$S_a = 208 \frac{N}{mm^2}$		$S_a = 196 \frac{N}{mm^2}$		$S_a = 183 \frac{N}{mm^2}$		$S_a = 171 \frac{N}{mm^2}$		$S_a = 159 \frac{N}{mm^2}$		$S_a = 147 \frac{N}{mm^2}$		$S_a = 134 \frac{N}{mm^2}$	
	N	N_ℓ	N	N_ℓ	N	N_ℓ	N	N_ℓ	N	N_ℓ	N	N_ℓ	N	N_ℓ
1	21,350	4.329	29,000	4.462	19,400	4.288	37,000	4.568	33,900	4.530	46,700	4.669	56,200	4.750
2	25,450	4.406	32,400	4.511	21,650	4.335	42,200	4.625	51,000	4.708	56,450	4.752	68,800	4.838
3	30,050	4.478	36,200	4.559	24,250	4.385	45,510	4.655	58,750	4.769	62,200	4.794	74,700	4.873
S_ℓ	0.051		0.049		0.035		0.044		0.124		0.064		0.063	
$\bar{N}_\ell$	4.442		4.511		4.360		4.616		4.669		4.738		4.820	
$S_\ell/\bar{N}_\ell$	1.15%		1.08%		0.81%		0.96%		2.66%		1.34%		1.31%	
$\bar{N}$	27,669		32,409		22,909		41,305		46,666		54,744		66,120	
N_{lower}	20,672		24,511		18,755		32,119		22,970		37,941		46,088	
N_{upper}	37,035		42,919		2,793		53,117		94,808		78,865		94,712	

	ALT. LOAD: 10 kN		ALT. LOAD: 9 kN		ALT. LOAD: 8 kN		ALT. LOAD: 7 kN		ALT. LOAD: 6 kN		ALT. LOAD: 5 kN	
	$S_a = 122 \frac{N}{mm^2}$		$S_a = 110 \frac{N}{mm^2}$		$S_a = 98 \frac{N}{mm^2}$		$S_a = 86 \frac{N}{mm^2}$		$S_a = 73 \frac{N}{mm^2}$		$S_a = 61 \frac{N}{mm^2}$	
	N	N_ℓ	N	N_ℓ	N	N_ℓ	N	N_ℓ	N	N_ℓ	N	N_ℓ
1	92,200	4.965	156,500	5.195	226,200	5.354	302,900	5.481	409,000	5.612	645,300	5.810
2	106,500	5.027	169,900	5.230	250,300	5.398	374,800	5.574	487,300	5.688	764,700	5.883
3	129,400	5.112	213,800	5.330	317,400	5.502	465,000	5.667	636,200	5.804	1,212,000	6.08
S_ℓ	0.074		0.070		0.076		0.093		0.097		0.142	
$\bar{N}_\ell$	5.035		5.252		5.418		5.574		5.701		5.926	
$S_\ell/\bar{N}_\ell$	1.47%		1.33%		1.40%		1.67%		1.70%		2.39%	
$\bar{N}$	108,310		178,512		261,818		374,973		502,728		842,688	
N_{lower}	71,005		119,734		169,559		220,352		288,527		374,516	
N_{upper}	165,468		266,552		404,276		638,091		874,607		1,899,022	

TABLE 9.1

Median Ranks

j*	Sample size n 1	2	3	4	5	6	7	8	9	10
1	.5000	.2929	.2063	.1591	.1294	.1091	.0943	.0830	.0741	.0670
2		.7071	.5000	.3864	.3147	.2655	.2295	.2021	.1806	.1632
3			.7937	.6136	.5000	.4218	.3648	.3213	.2871	.2594
4				.8409	.6853	.5782	.5000	.4404	.3935	.3557
5					.8706	.7345	.6352	.5596	.5000	.4519
6						.8909	.7705	.6787	.6065	.5481
7							.9057	.7979	.7129	.6443
8								.9170	.8194	.7406
9									.9259	.8368
10										.9330

* Order number.

TABLE 9.2

The Correlation Coefficient

Values of the Correlation Coefficient for Different Levels of Significance

n	·1	·05	·02	·01	·001	n	·1	·05	·02	·01	·001
1	·98769	·99692	·999507	·999877	·9999988	16	·4000	·4683	·5425	·5897	·7084
2	·90000	·95000	·98000	·990000	·99900	17	·3887	·4555	·5285	·5751	·6932
3	·8054	·8783	·93433	·95873	·99116	18	·3783	·4438	·5155	·5614	·6787
4	·7293	·8114	·8822	·91720	·97406	19	·3687	·4329	·5034	·5487	·6652
5	·6694	·7545	·8329	·8745	·95074	20	·3598	·4227	·4921	·5368	·6524
6	·6215	·7067	·7887	·8343	·92493	25	·3233	·3809	·4451	·4869	·5974
7	·5822	·6664	·7498	·7977	·8982	30	·2960	·3494	·4093	·4487	·5541
8	·5494	·6319	·7155	·7646	·8721	35	·2746	·3246	·3810	·4182	·5189
9	·5214	·6021	·6851	·7348	·8471	40	·2573	·3044	·3578	·3932	·4896
10	·4973	·5760	·6581	·7079	·8233	45	·2428	·2875	·3384	·3721	·4648
11	·4762	·5529	·6339	·6835	·8010	50	·2306	·2732	·3218	·3541	·4433
12	·4575	·5324	·6120	·6614	·7800	60	·2108	·2500	·2948	·3248	·4078
13	·4409	·5139	·5923	·6411	·7603	70	·1954	·2319	·2737	·3017	·3799
14	·4259	·4973	·5742	·6226	·7420	80	·1829	·2172	·2565	·2830	·3568
15	·4124	·4821	·5577	·6055	·7246	90	·1726	·2050	·2422	·2673	·3375
						100	·1638	·1946	·2301	·2540	·3211

TABLE 9.3

VALUES OF t.

Degrees of Freedom	$t_{0.95}$	$t_{0.975}$	$t_{0.9875}$	$t_{0.995}$	$t_{0.9975}$
1	6.31	12.7	25.5	63.7	127
2	2.92	4.30	6.21	9.92	14.1
3	2.35	3.18	4.18	5.84	7.45
4	2.13	2.78	3.50	4.60	5.60
5	2.01	2.57	3.16	4.03	4.77
6	1.94	2.45	2.97	3.71	4.32
7	1.89	2.36	2.84	3.50	4.03
8	1.86	2.31	2.75	3.36	3.83
9	1.83	2.26	2.69	3.25	3.69
10	1.81	2.23	2.63	3.17	3.58
11	1.80	2.20	2.59	3.11	3.50
12	1.78	2.18	2.56	3.05	3.43
13	1.77	2.16	2.53	3.01	3.37
14	1.76	2.14	2.51	2.98	3.33
15	1.75	2.13	2.49	2.95	3.29
16	1.75	2.12	2.47	2.92	3.25
17	1.74	2.11	2.46	2.90	3.22
18	1.73	2.10	2.45	2.88	3.20
19	1.73	2.09	2.43	2.86	3.17
20	1.72	2.09	2.42	2.85	3.15
21	1.72	2.08	2.41	2.83	3.14
22	1.72	2.07	2.41	2.82	3.12
23	1.71	2.07	2.40	2.81	3.10
24	1.71	2.06	2.39	2.80	3.09
25	1.71	2.06	2.38	2.79	3.08
26	1.71	2.06	2.38	2.78	3.07
27	1.70	2.05	2.37	2.77	3.06
28	1.70	2.05	2.37	2.76	3.05
29	1.70	2.05	2.36	2.76	3.04
30	1.70	2.04	2.36	2.75	3.03
40	1.68	2.02	2.33	2.70	2.97
60	1.67	2.00	2.30	2.66	2.91
120	1.66	1.98	2.27	2.62	2.86
∞	1.64	1.96	2.24	2.58	2.81
Degrees of Freedom	$t_{0.05}$	$t_{0.025}$	$t_{0.0125}$	$t_{0.005}$	$t_{0.0025}$

When the table is read from the foot, the tabled values are to be prefixed with a negative sign. Interpolation should be performed using the reciprocals of the degrees of freedom.

TABLE 9-4

k FACTORS* FOR S-N CURVES

n \ p	75	90	95	99	99.9	75	90	95	99	99.9
	$\gamma = 0.50$					$\gamma = 0.75$				
3	0.773	1.498	1.939	2.765	3.688	1.464	2.501	3.152	4.396	5.805
4	0.739	1.419	1.830	2.601	3.464	1.256	2.134	2.680	3.726	4.910
5	0.722	1.382	1.780	2.526	3.362	1.152	1.961	2.463	3.421	4.507
6	0.712	1.360	1.750	2.483	3.304	1.087	1.860	2.336	3.243	4.273
7	0.705	1.346	1.732	2.455	3.265	1.043	1.791	2.250	3.126	4.118
8	0.701	1.337	1.719	2.436	3.239	1.010	1.740	2.190	3.042	4.008
9	0.698	1.329	1.709	2.421	3.220	0.984	1.702	2.141	2.977	3.924
10	0.694	1.324	1.702	2.411	3.205	0.964	1.671	2.103	2.927	3.858
11	0.693	1.320	1.696	2.402	3.193	0.947	1.646	2.073	2.885	3.804
12	0.691	1.316	1.691	2.395	3.183	0.933	1.624	2.048	2.851	3.760
13	0.690	1.313	1.687	2.388	3.175	0.919	1.606	2.026	2.822	3.722
14	0.689	1.311	1.684	2.384	3.168	0.909	1.591	2.007	2.796	3.690
15	0.688	1.308	1.680	2.379	3.163	0.899	1.577	1.991	2.776	3.661
16	0.686	1.307	1.678	2.376	3.157	0.891	1.566	1.977	2.756	3.637
17	0.686	1.305	1.676	2.373	3.153	0.883	1.554	1.964	2.739	3.615
18	0.685	1.303	1.674	2.370	3.150	0.876	1.544	1.951	2.723	3.595
19	0.684	1.302	1.672	2.367	3.146	0.870	1.536	1.942	2.710	3.577
20	0.684	1.301	1.671	2.366	3.143	0.865	1.528	1.933	2.697	3.561
21	0.683	1.300	1.670	2.364	3.140	0.859	1.520	1.923	2.686	3.545
22	0.683	1.299	1.668	2.361	3.138	0.854	1.514	1.916	2.675	3.532
23	0.683	1.299	1.668	2.360	3.136	0.849	1.508	1.907	2.665	3.520
24	0.682	1.298	1.667	2.358	3.134	0.845	1.502	1.901	2.656	3.509
25	0.682	1.297	1.666	2.357	3.132	0.842	1.496	1.895	2.647	3.497
	$\gamma = 0.90$					$\gamma = 0.95$				
3	2.602	4.258	5.310	7.340	9.651	3.804	6.158	7.655	10 552	13.857
4	1.972	3.187	3.957	5.437	7.128	2.619	4.163	5.145	7.042	9.215
5	1.698	2.742	3.400	4.666	6.112	2.149	3.407	4.202	5.741	7.501
6	1.540	2.494	3.091	4.242	5.556	1.895	3.006	3.707	5.062	6.612
7	1.435	2.333	2.894	3.972	5.201	1.732	2.755	3.399	4.641	6.061
8	1.360	2.219	2.755	3.783	4.955	1.617	2.582	3.188	4.353	5.686
9	1.302	2.133	2.649	3.641	4.772	1.532	2.454	3.031	4.143	5.414
10	1.257	2.065	2.568	3.532	4.629	1.465	2.355	2.911	3 981	5.203
11	1.219	2.012	2.503	3.444	4.515	1.411	2.275	2.815	3.852	5.036
12	1.188	1.966	2.448	3.371	4.420	1.366	2.210	2.736	3.747	4.900
13	1.162	1.928	2.403	3.310	4.341	1.329	2.155	2.670	3.659	4.787
14	1.139	1.895	2.363	3.257	4.274	1.296	2.108	2.614	3.585	4.690
15	1.119	1.866	2.329	3.212	4.215	1.268	2.068	2.566	3.520	4.607
16	1.101	1.842	2.299	3.172	4.164	1.242	2.032	2.523	3.463	4.534
17	1.085	1.820	2.272	3.136	4.118	1.220	2.001	2.486	3.415	4.471
18	1.071	1.800	2.249	3.106	4.078	1.200	1.974	2.453	3.370	4.415
19	1.058	1.781	2.228	3.078	4.041	1.183	1.949	2.423	3.331	4.364
20	1.046	1.765	2.208	3.052	4.009	1.167	1.926	2.396	3.295	4.319
21	1.035	1.750	2.190	3.028	3.979	1.152	1.905	2.371	3.262	4.276
22	1.025	1.736	2.174	3.007	3.952	1.138	1.887	2.350	3.233	4.238
23	1.016	1.724	2.159	2.987	3.927	1.126	1.869	2.329	3.206	4.204
24	1.007	1.712	2.145	2.969	3.904	1.114	1.853	2.309	3.151	4.171
25	0.999	1.702	2.132	2.952	3.882	1.103	1.838	2.292	3.158	4 143

* In which:
n = sample size,
p = per cent survival, and
γ = confidence level.

TABLE 9.5

EQUATIONS OF SURVIVAL CURVES OF ϕ10-BOLTS

(γ = 0.90)

Mean Load (kN)	Mean Stress (N/mm^2)	Percent Survival (p %)	Equation of the best fit line $\frac{S_a}{S_t}$ =	Correlation Coefficient (r)
10	172	10	- 0.1039 log N + 0.8120	0.971
10	172	50	- 0.1106 log N + 0.8118	0.989
10	172	90	- 0.1184 log N + 0.8121	0.987
15	258	10	- 0.1075 log N + 0.7894	0.975
15	258	50	- 0.1103 log N + 0.7664	0.981
15	258	90	- 0.1134 log N + 0.7662	0.990
20	345	10	- 0.0998 log N + 0.7226	0.991
20	345	50	- 0.1066 log N + 0.7380	0.989
20	345	90	- 0.1142 log N + 0.7543	0.994

TABLE 10.1

COLLECTED RESULTS OF ANALYSIS OF FATIGUE DATA FOR SCREWED BARS AND BOLTS

Symbol	SPECIMEN TYPE	ϕ12-SCREWED BAR				ϕ12-BOLT	ϕ10-BOLT		
	Test Machine	Avery		Vibraphore		Avery	Vibraphore		
S_t	Tensile Strength (N/mm^2)	538		538		981	590		
F_m	Mean Load (kN)	10	20	10	20	20	10	15	20
S_m	Mean Stress (N/mm^2)	134	268	134	268	245	172	259	345
B	Equation Constant	34,240	70,748	28,057	58,986	28,518	45,911	42,588	46,668
S_e exp. (10^7)	Experimental Fatigue Limit at 10^7c. (N/mm^2)	87	54	107	87	49	47.4	49.1	51.7
S_e th. (10^7)	Theoretical Fatigue Limit at 10^7c. (N/mm^2)	88.5	55.9	117.7	89	46.3	57.5	51.4	48.8
S_e th. (10^8)	Theoretical Fatigue Limit at 10^8c. (N/mm^2)	75	45.7	102	75.9	38.4	47.4	42.7	40.7
S_e th. ($N \to \infty$)	Theoretical Fatigue Limit as $N \to \infty$ (N/mm^2)	36.3	20	52.7	36.8	17.5	21.2	19.4	18.8

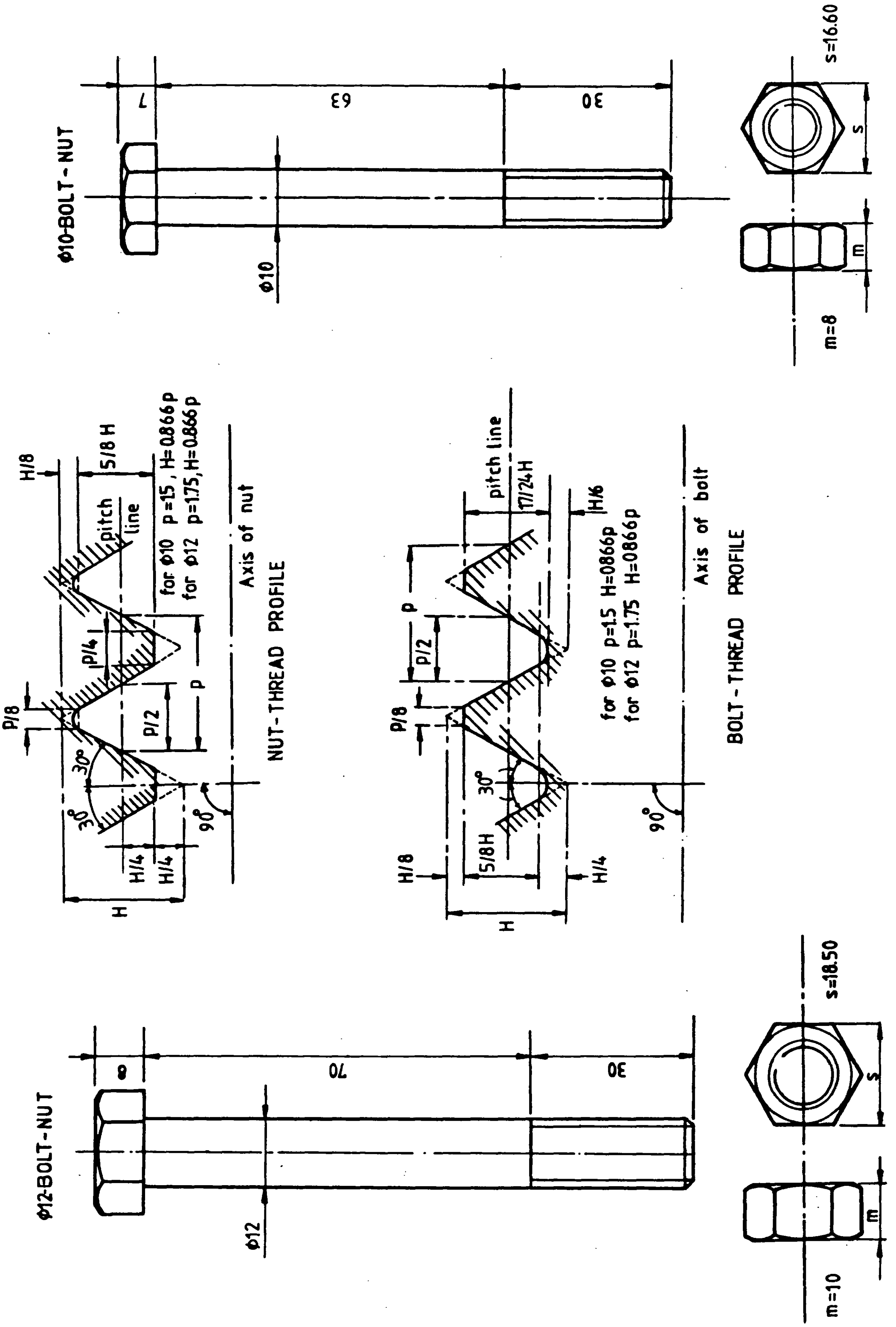


FIG. 3.1 DETAILS OF GEOMETRY OF BOLTS

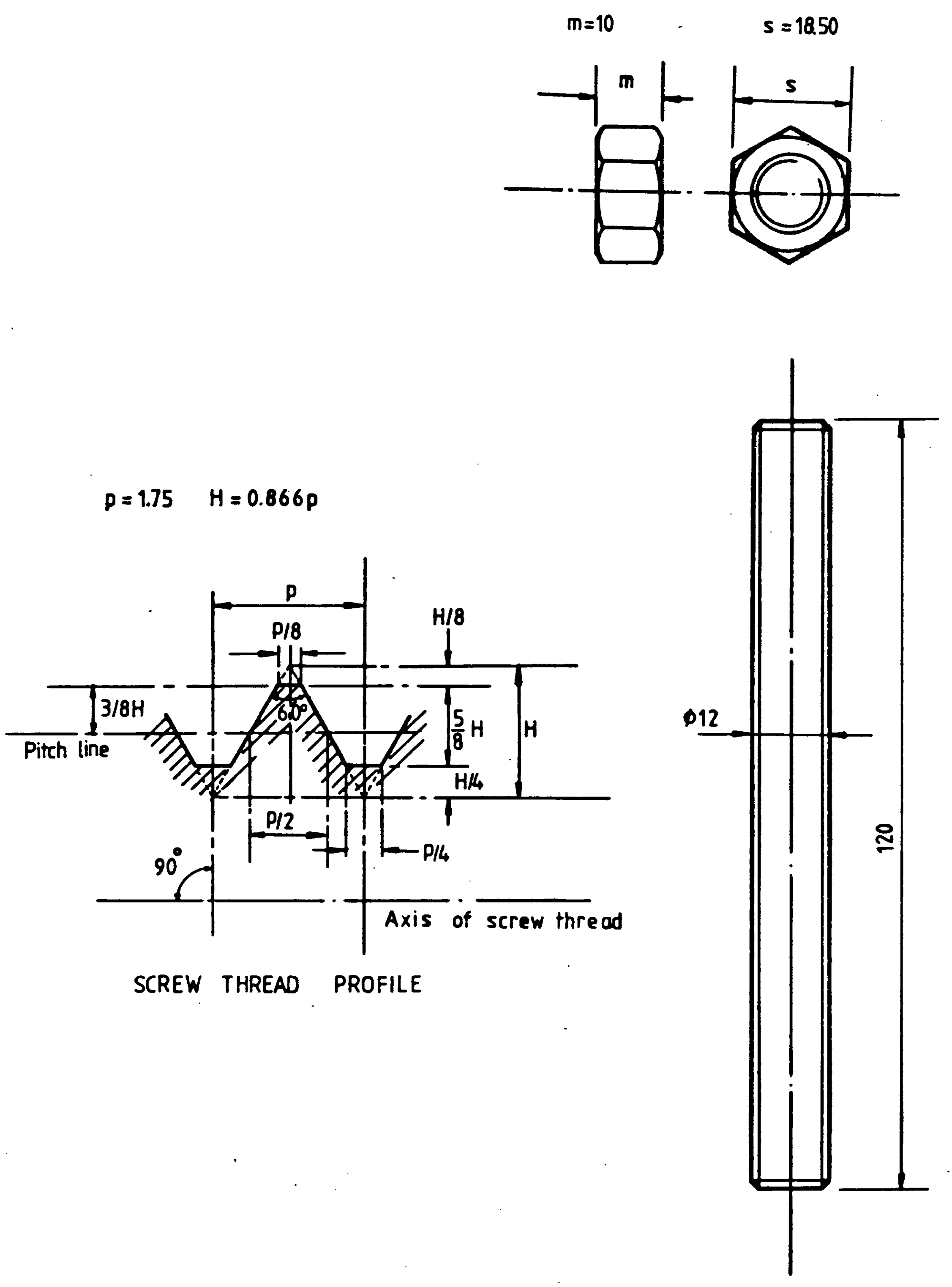
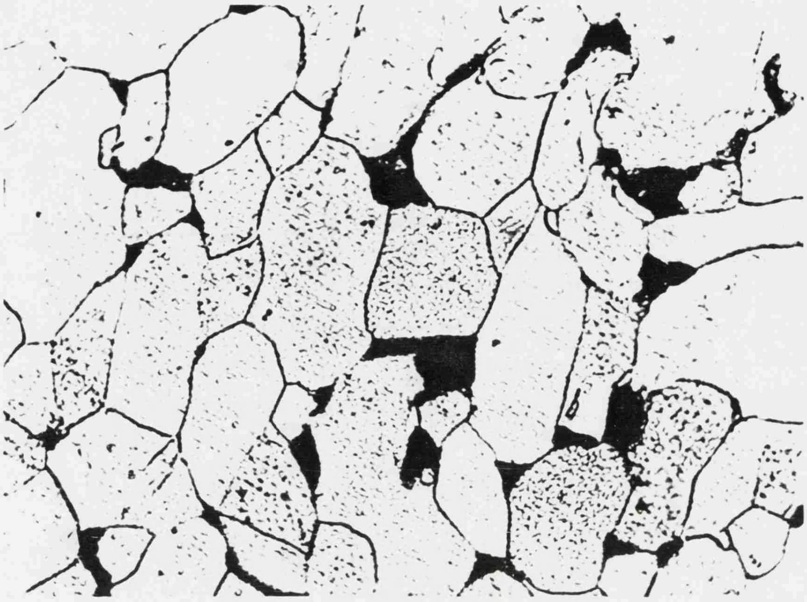
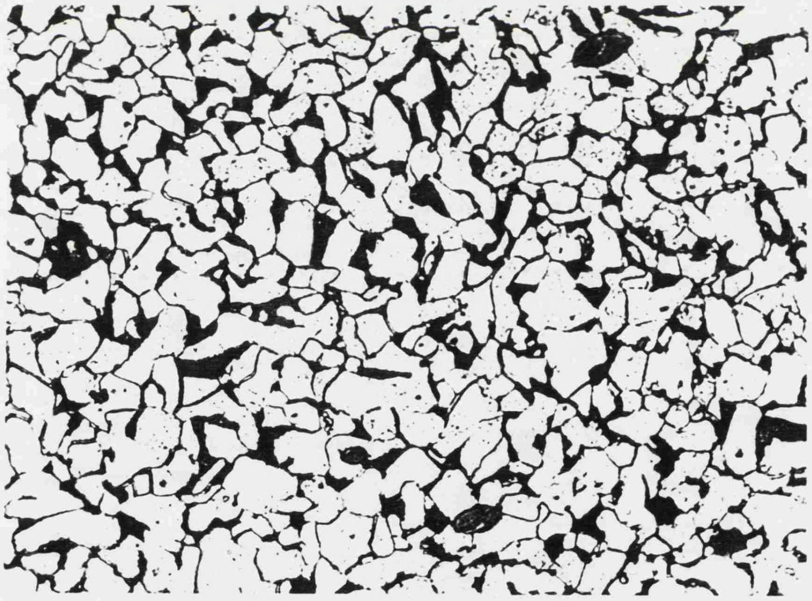


FIG.3 2 DETAILES OF GEOMETRY OF SCR.-BAR

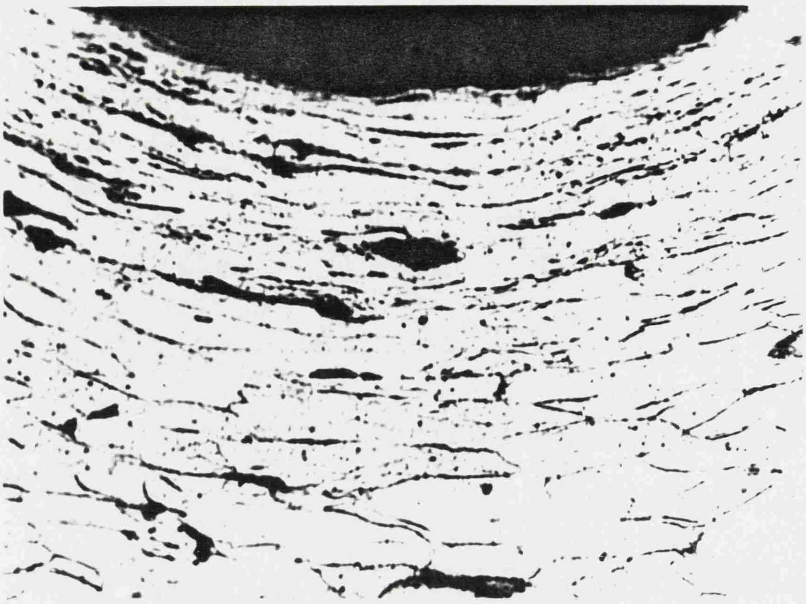


Transverse section of Ø12 screwed bar– X410

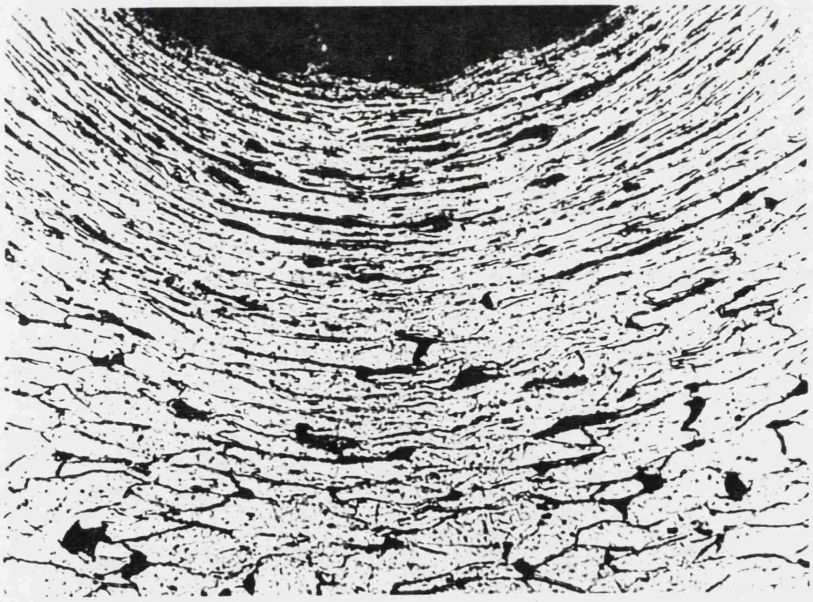


Transverse section of Ø10 bolt– X410

FIG.33 MICROSTRUCTURES OF SCREWED BAR AND BOLT



Longitudinal section of Ø12 screwed bar – X410



Longitudinal section of Ø10 bolt – X410

FIG. 3.4 MICROSTRUCTURES OF SCREWED BAR AND BOLT

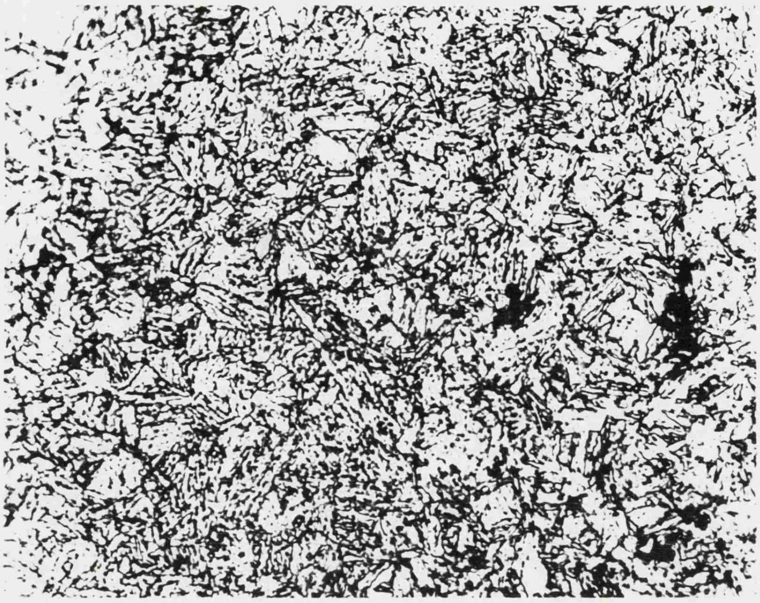


FIG.3 5 Transverse section of ϕ12 bolt – X410

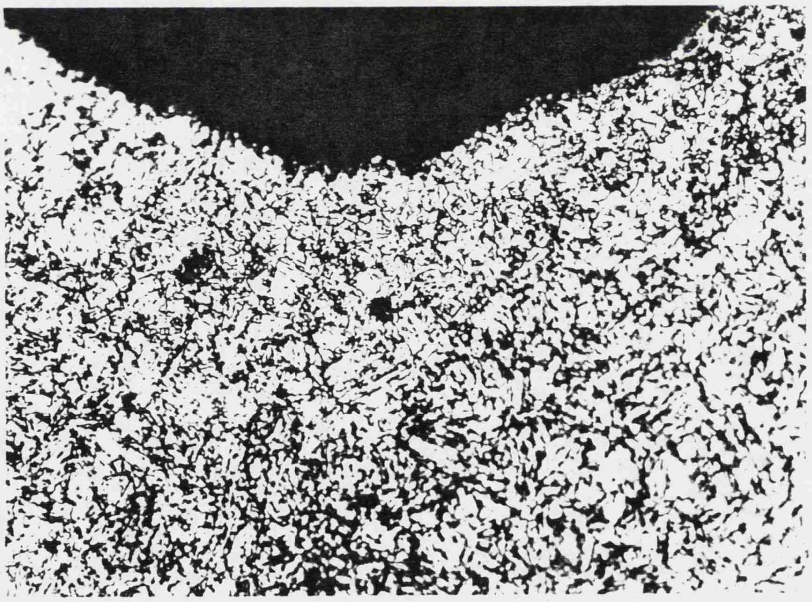


FIG.3 6 Longitudinal section of ϕ12 bolt _ X410

FIGS. 3.5 & 3.6 MICROSTRUCTURES OF ϕ12 BOLT

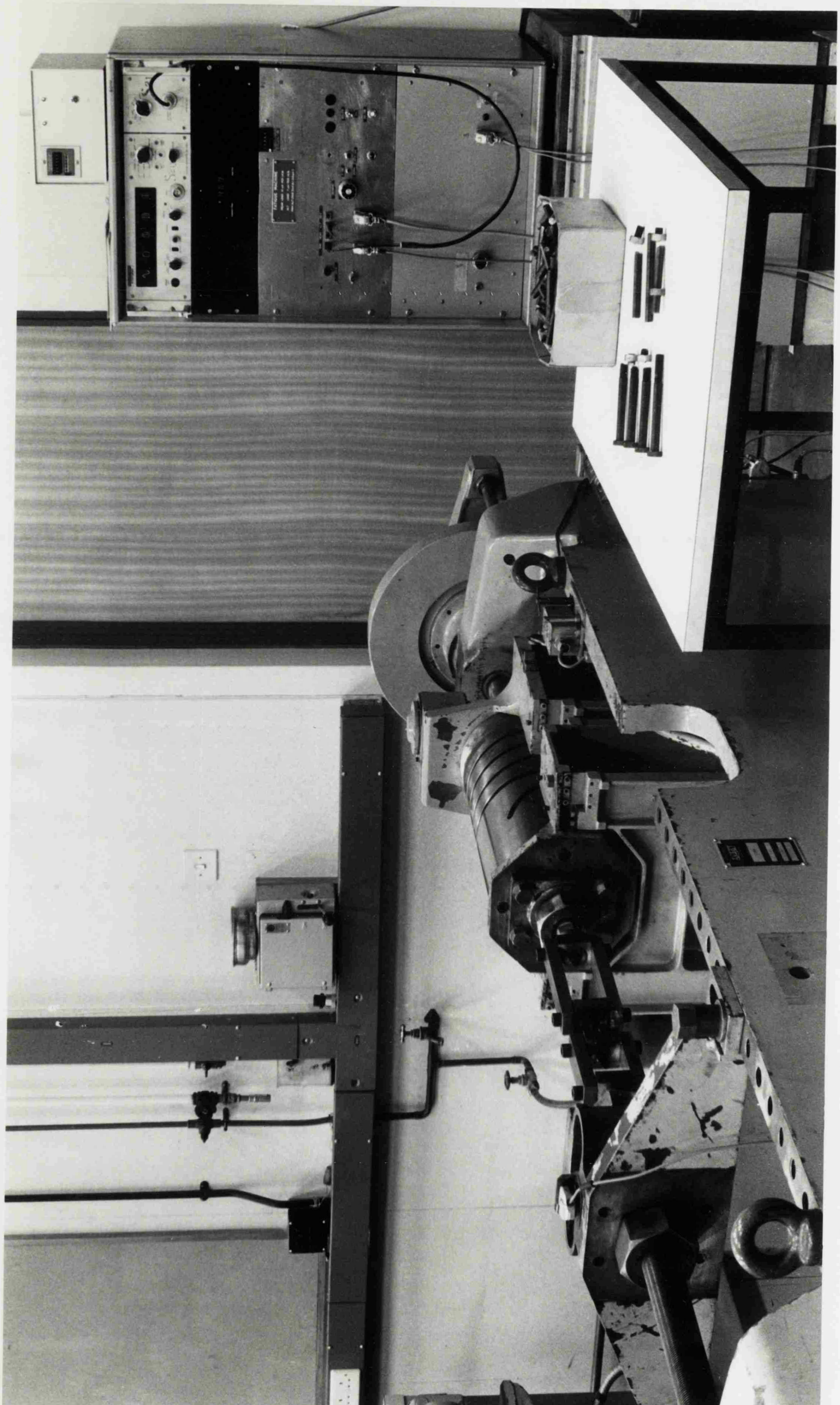


FIG.4.1 AVERY-SCHENCK PULSATOR

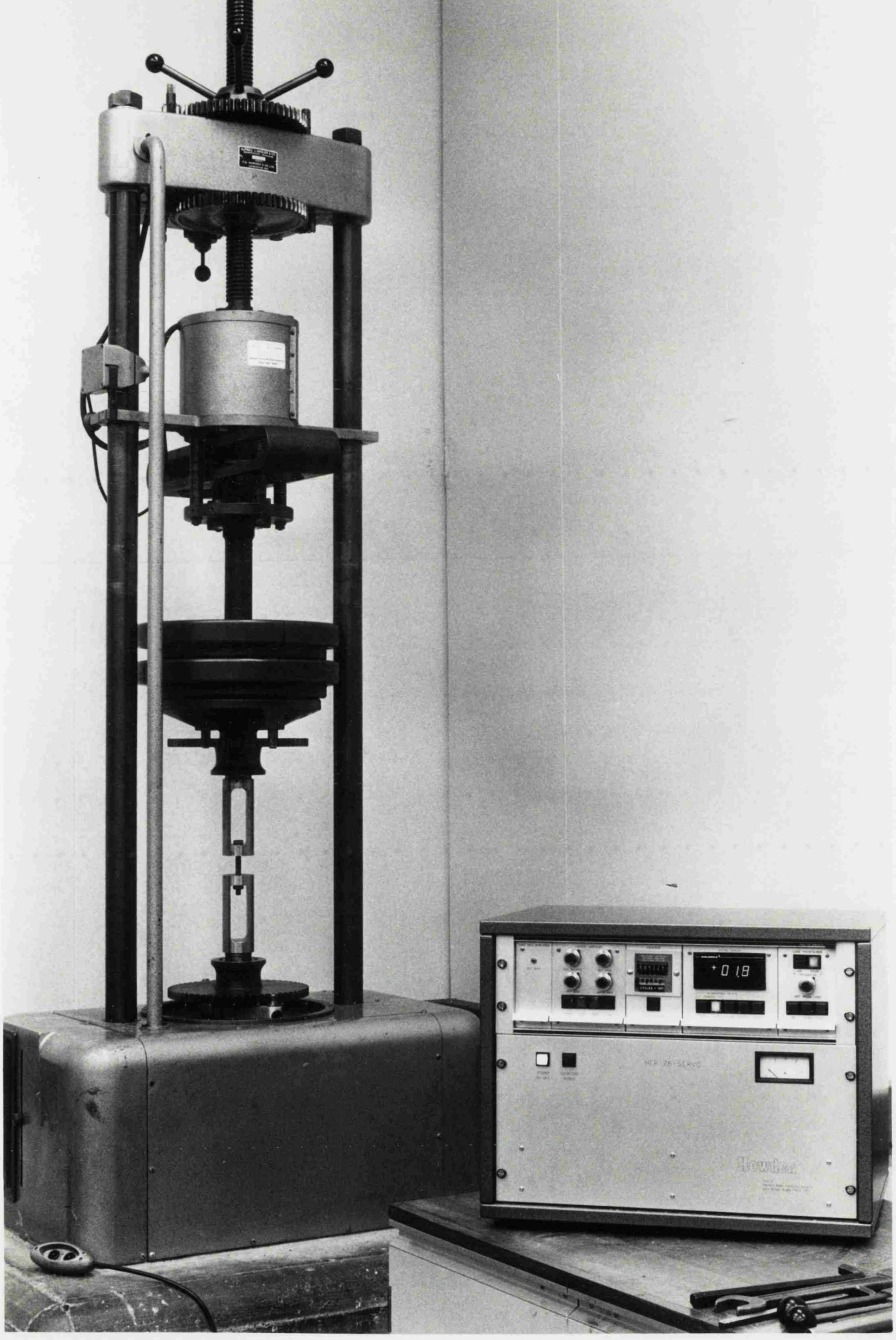


FIG. 4.2 AMSLER VIBRAPHORE

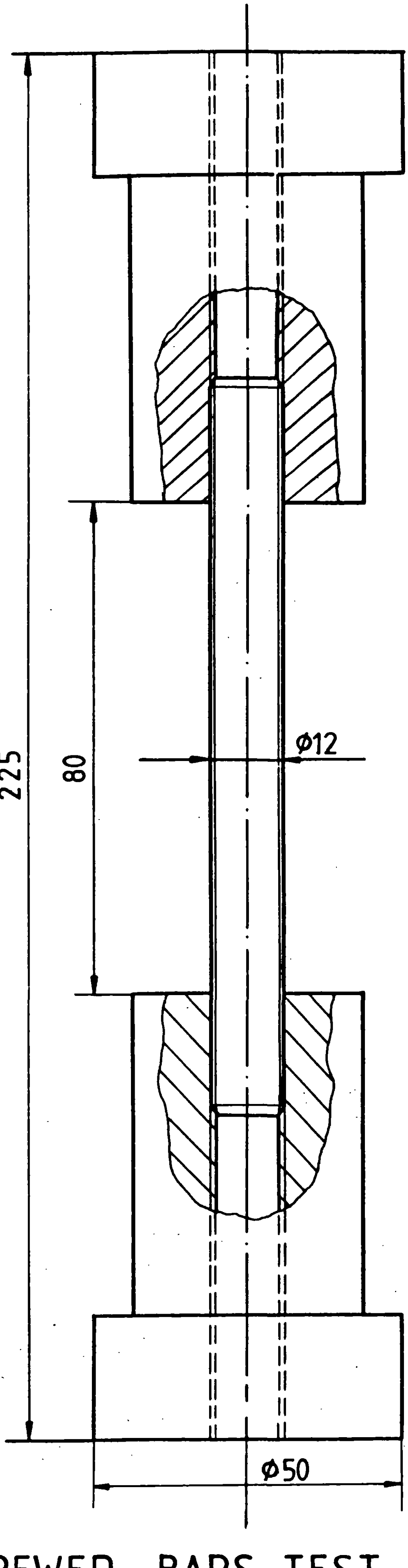


FIG.4.3 SCREWED BARS TEST RIG
FOR AVERY SCHENCK

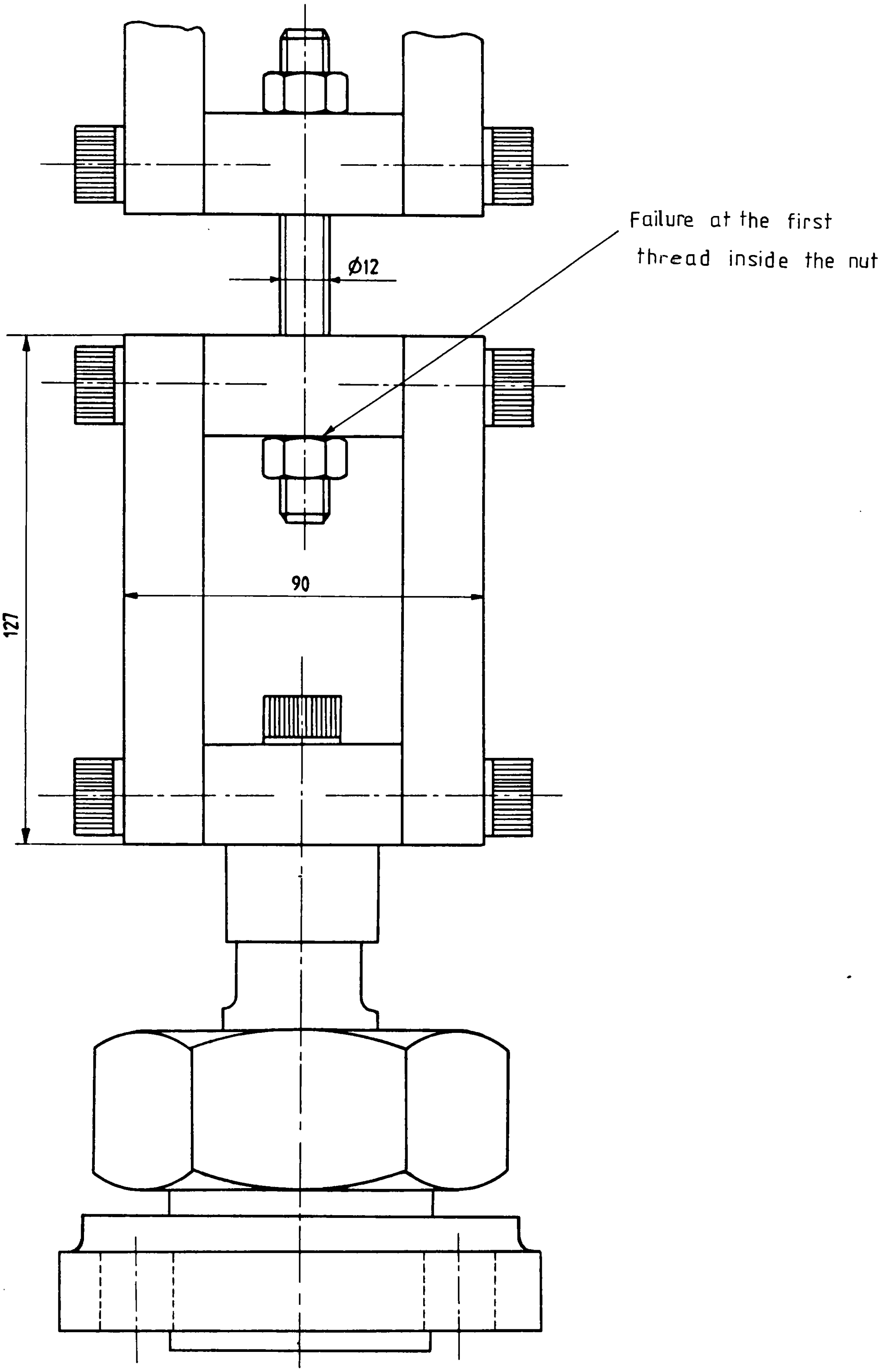


FIGURE 4.4
SCREWED BARS TEST RIG FOR AVERY FATIGUE MACHINE

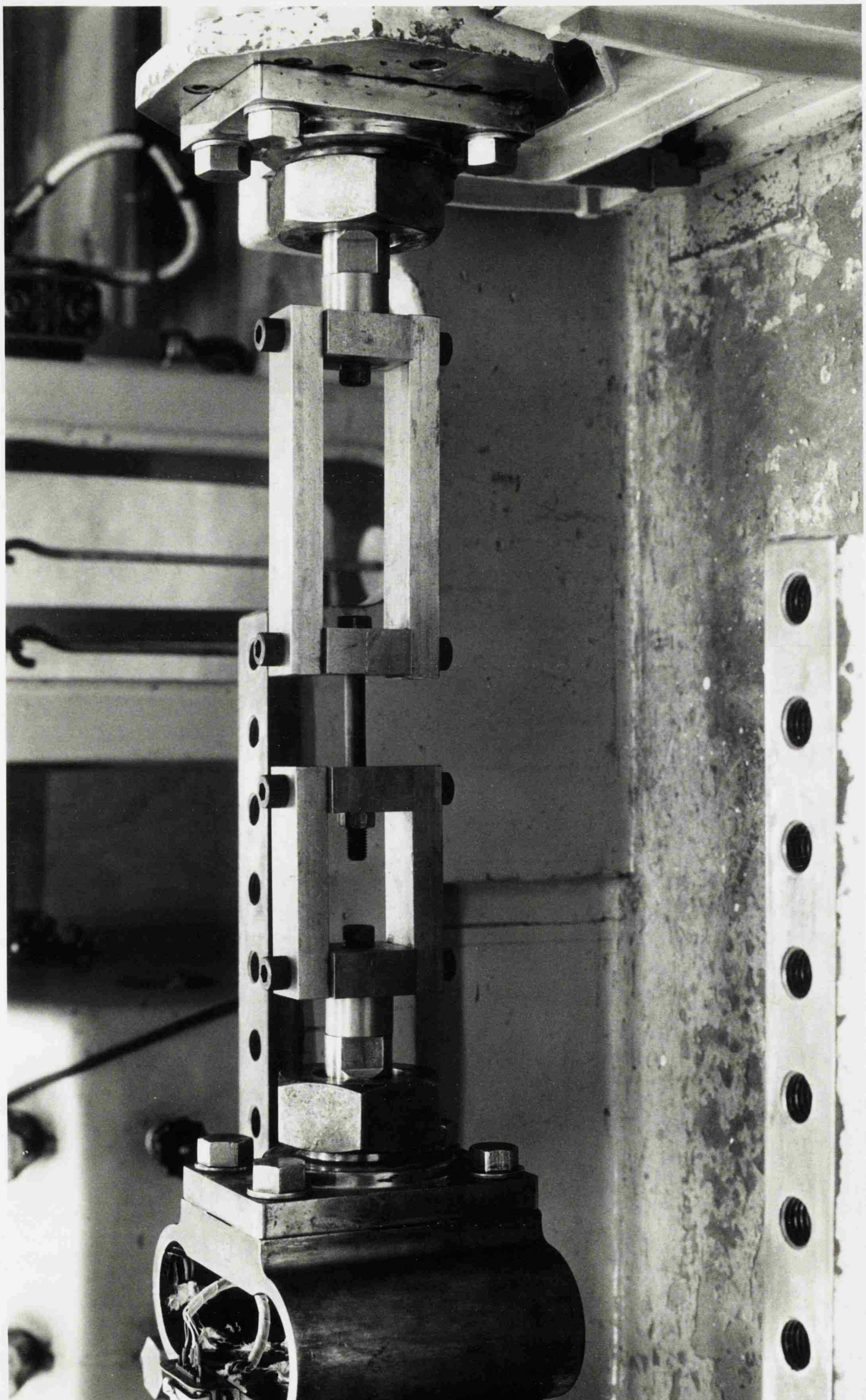


FIG. 4.4* AVERY TEST ADAPTERS

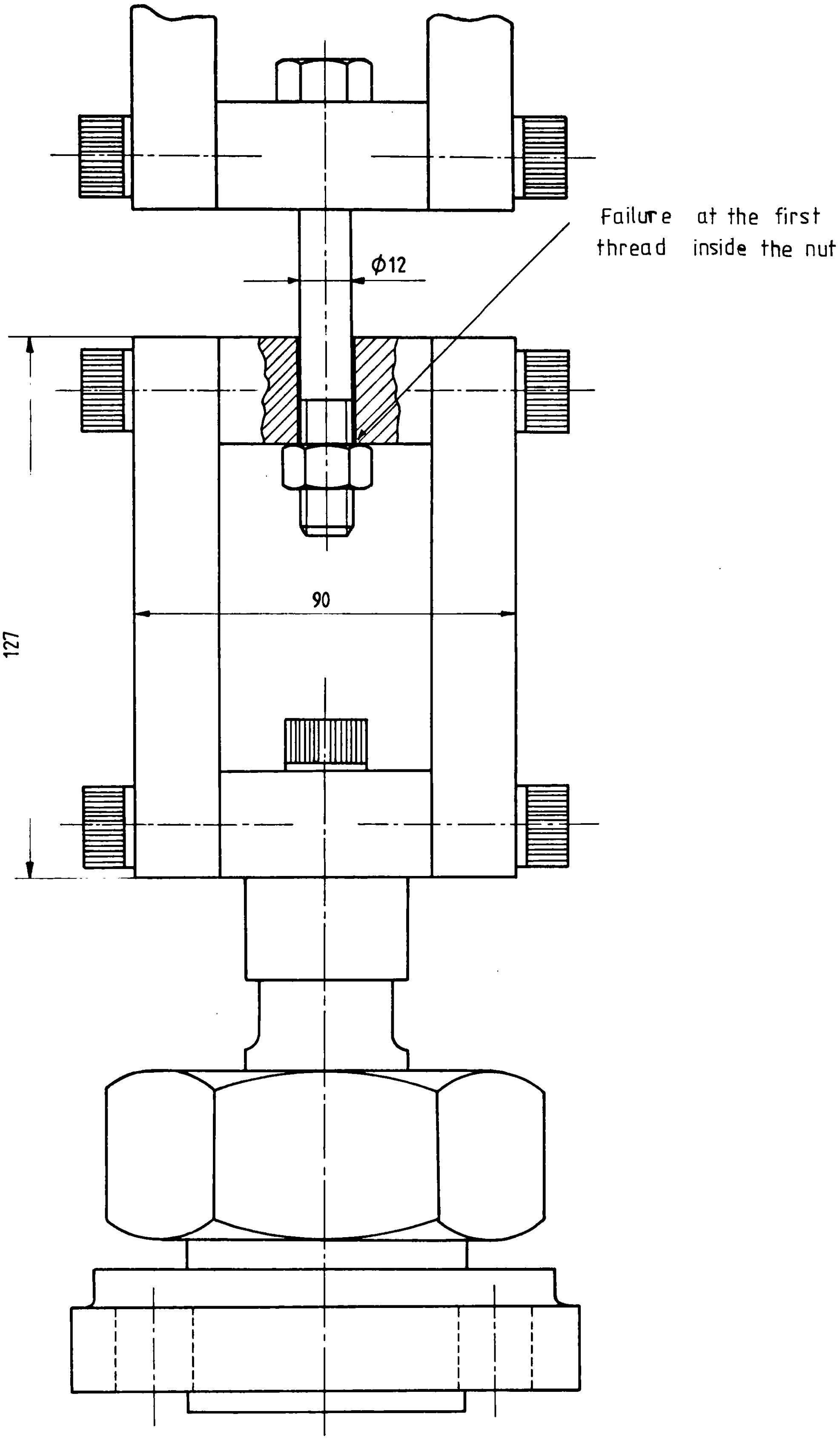


FIGURE 4_5a

TENSION BOLTS TEST RIG FOR AVERY FATIGUE MACHINE

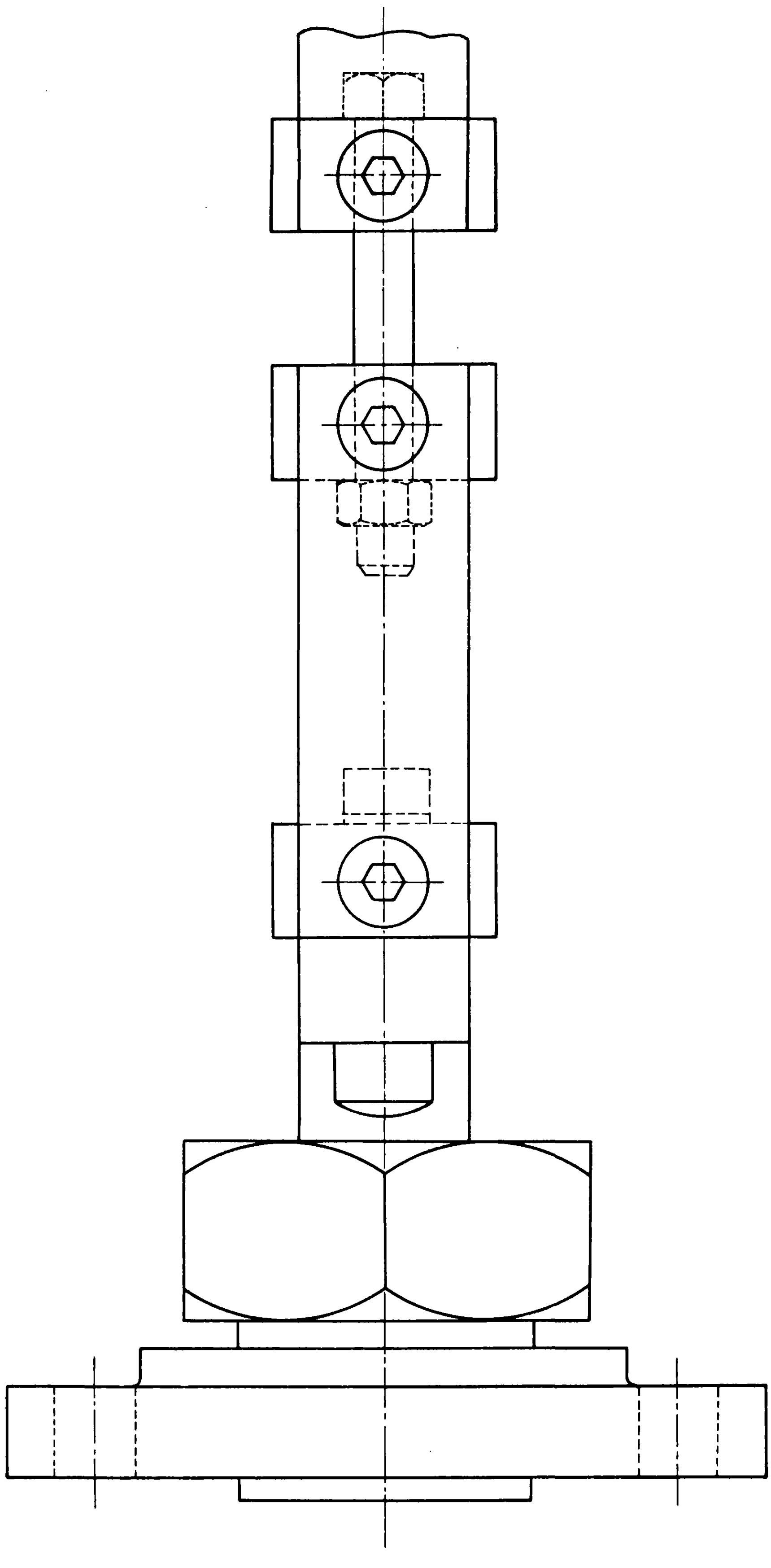


FIGURE 4_5b

SIDE VIEW OF FIGURE 4_5a

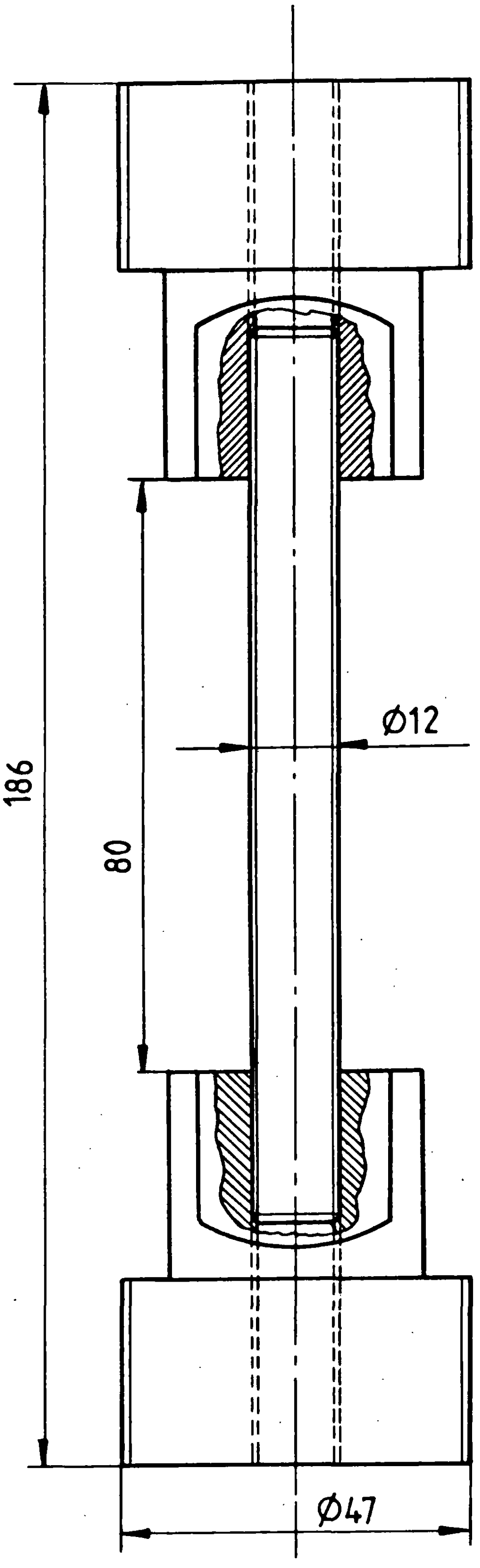


FIG.4.6 SCREWED BARS TEST RIG
FOR AMSLER VIBRAPHORE

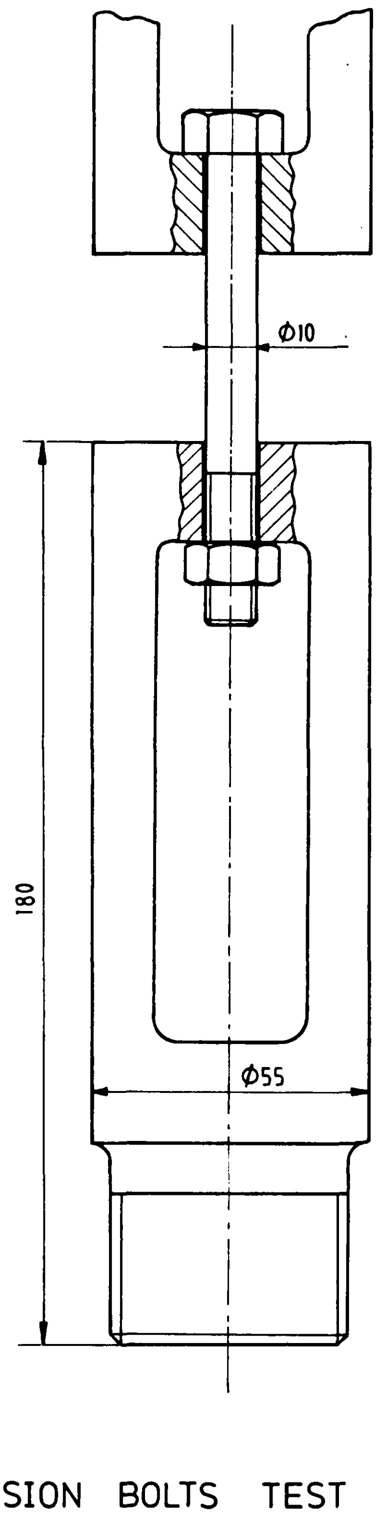


FIGURE 4_7 TENSION BOLTS TEST RIG
FOR AMSLER – VIBRAPHORE

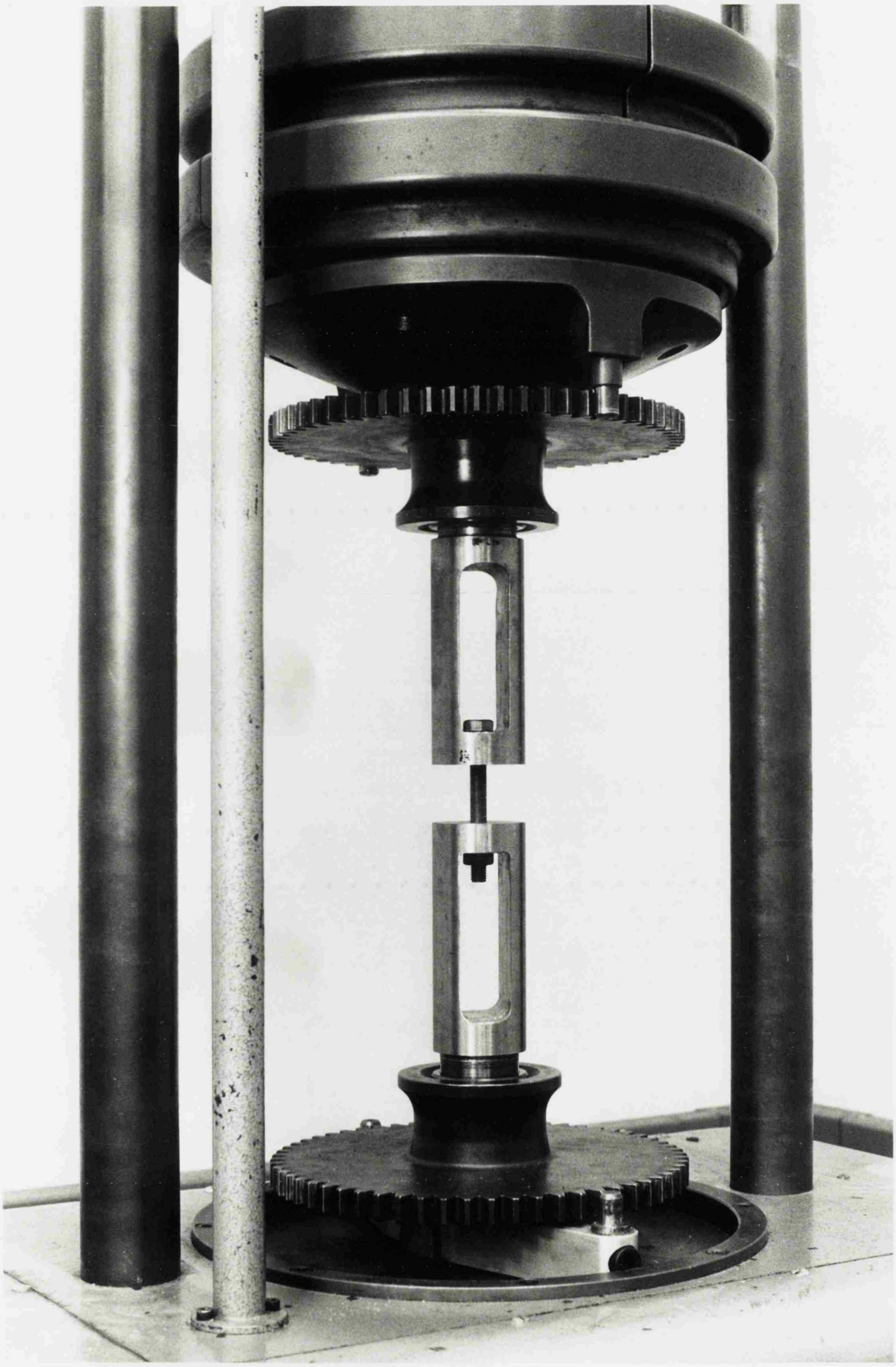


FIG. 4.7* VIBRAPHORE TEST ADAPTERS

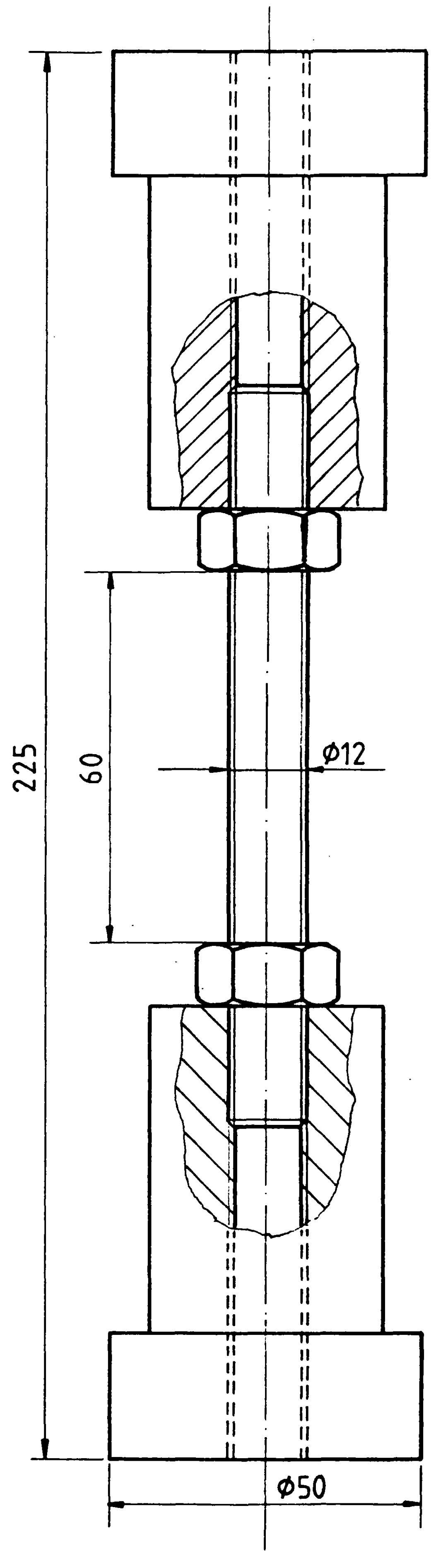


FIG.5.1 SCREWED BAR LOADED WITH NUTS IN AVERY-SCHENCK

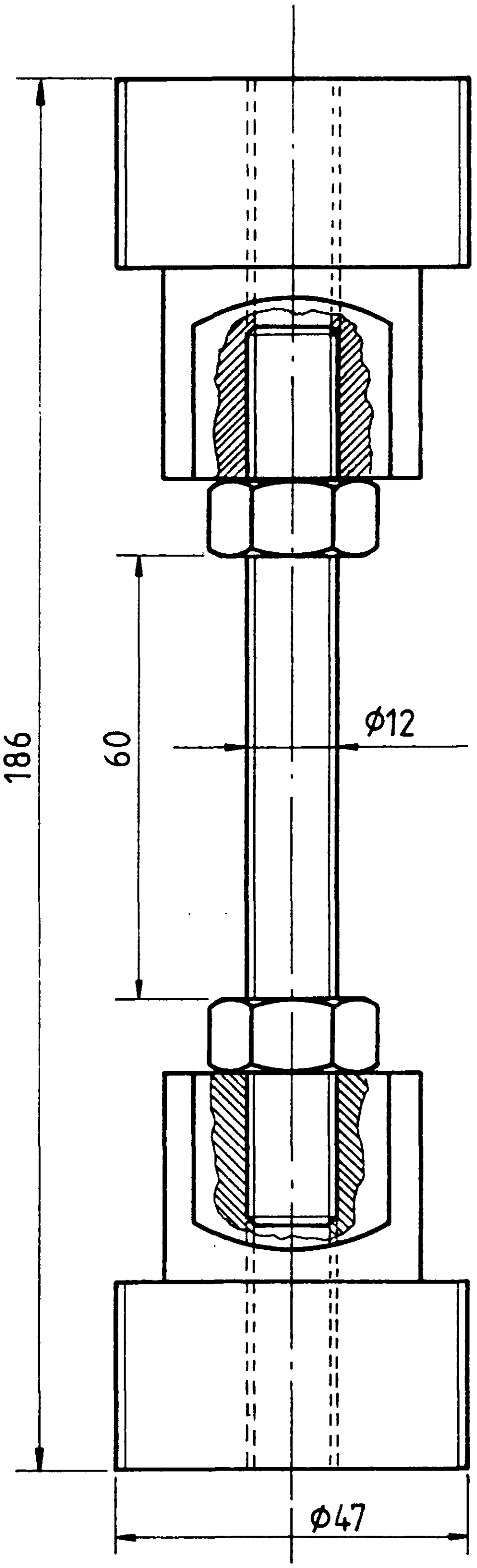


FIG.5.2 SCREWED BAR LOADED WITH NUTS IN VIBRAPHORE

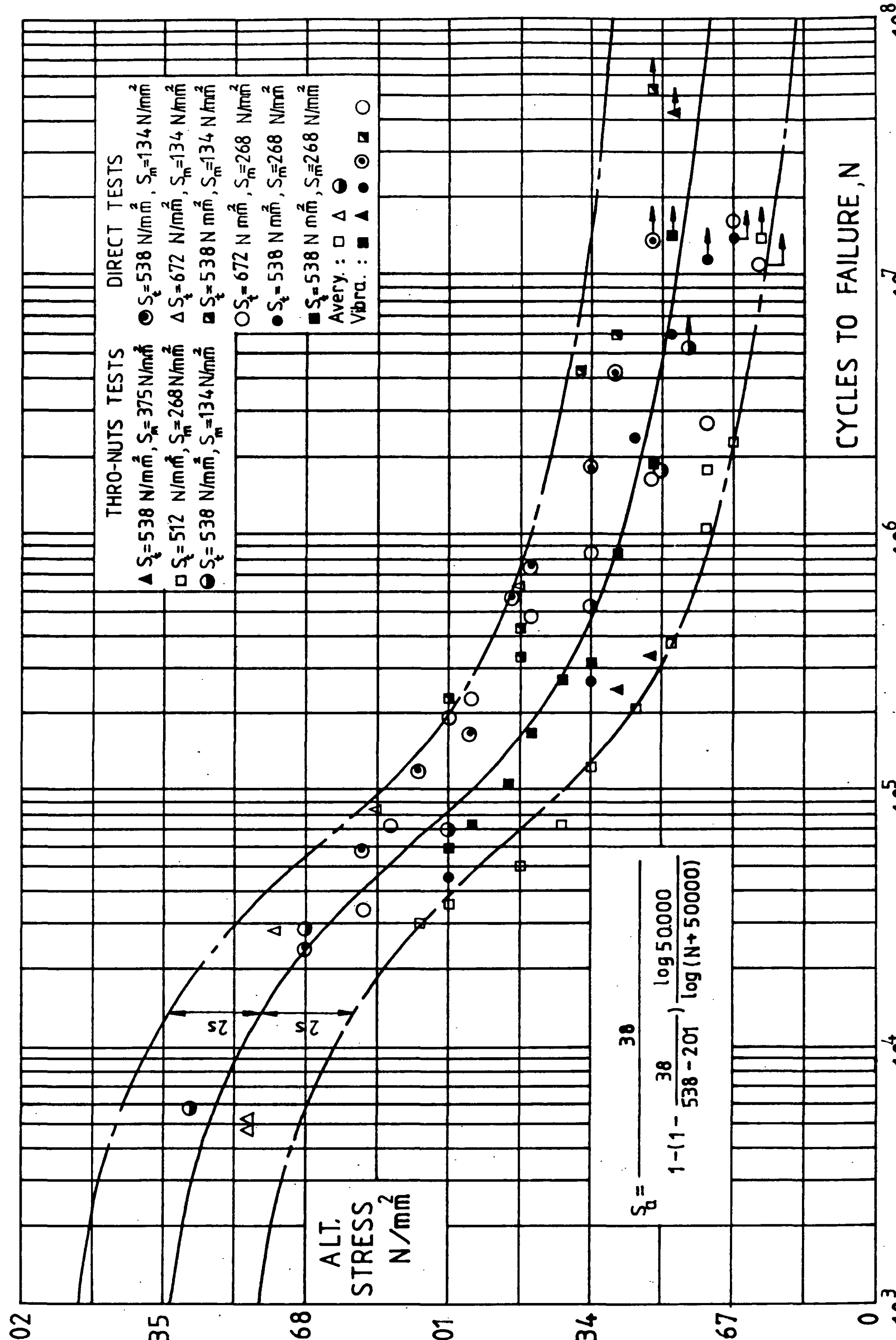


FIG. 5.3 COLLECTED FATIGUE TEST RESULTS OF THE Ø12-SCREWED BAR

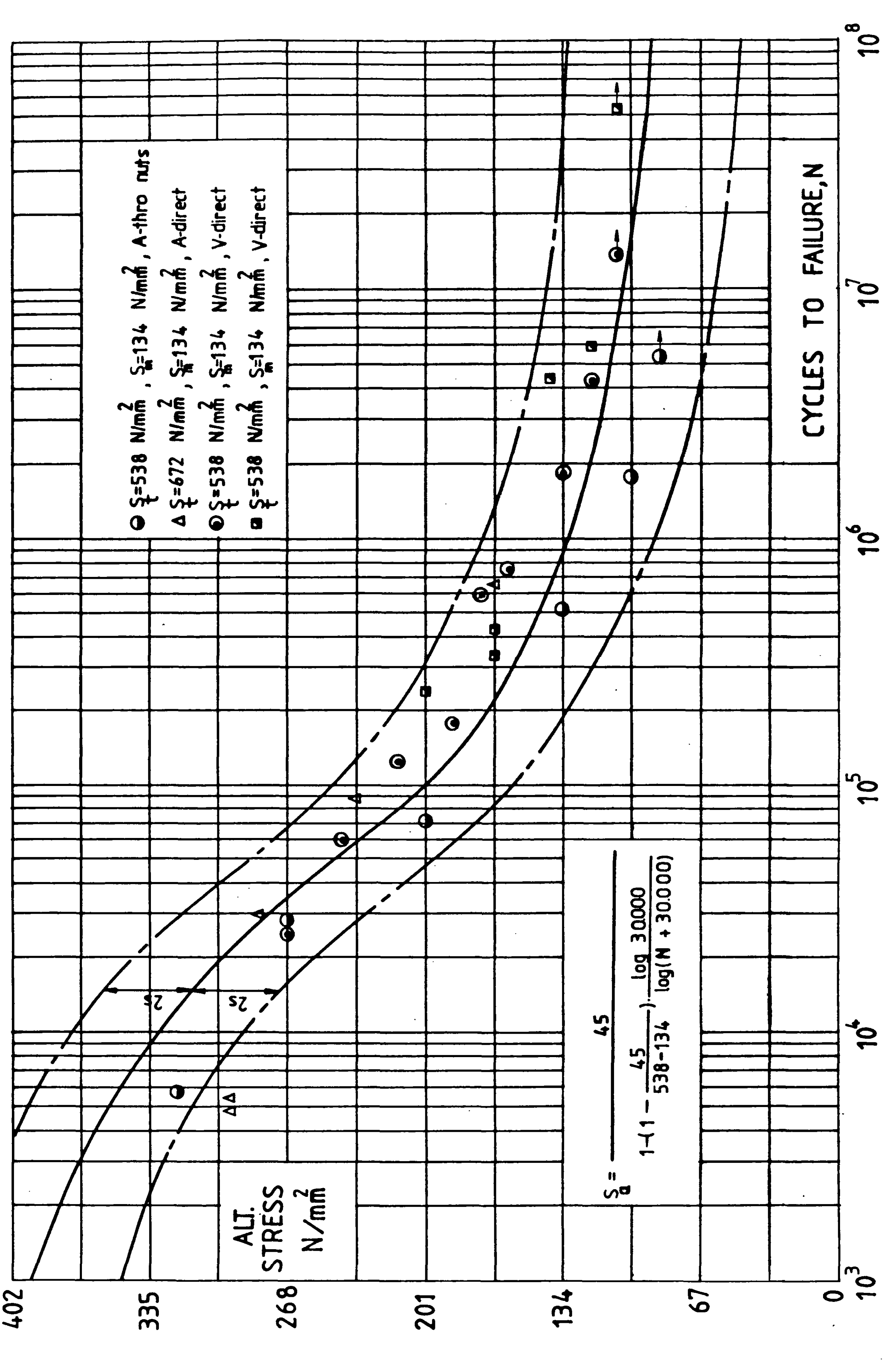


FIG. 5.4 FATIGUE TEST RESULTS OF THE Ø12-SCREWED BAR AT 10 KN MEAN LOAD

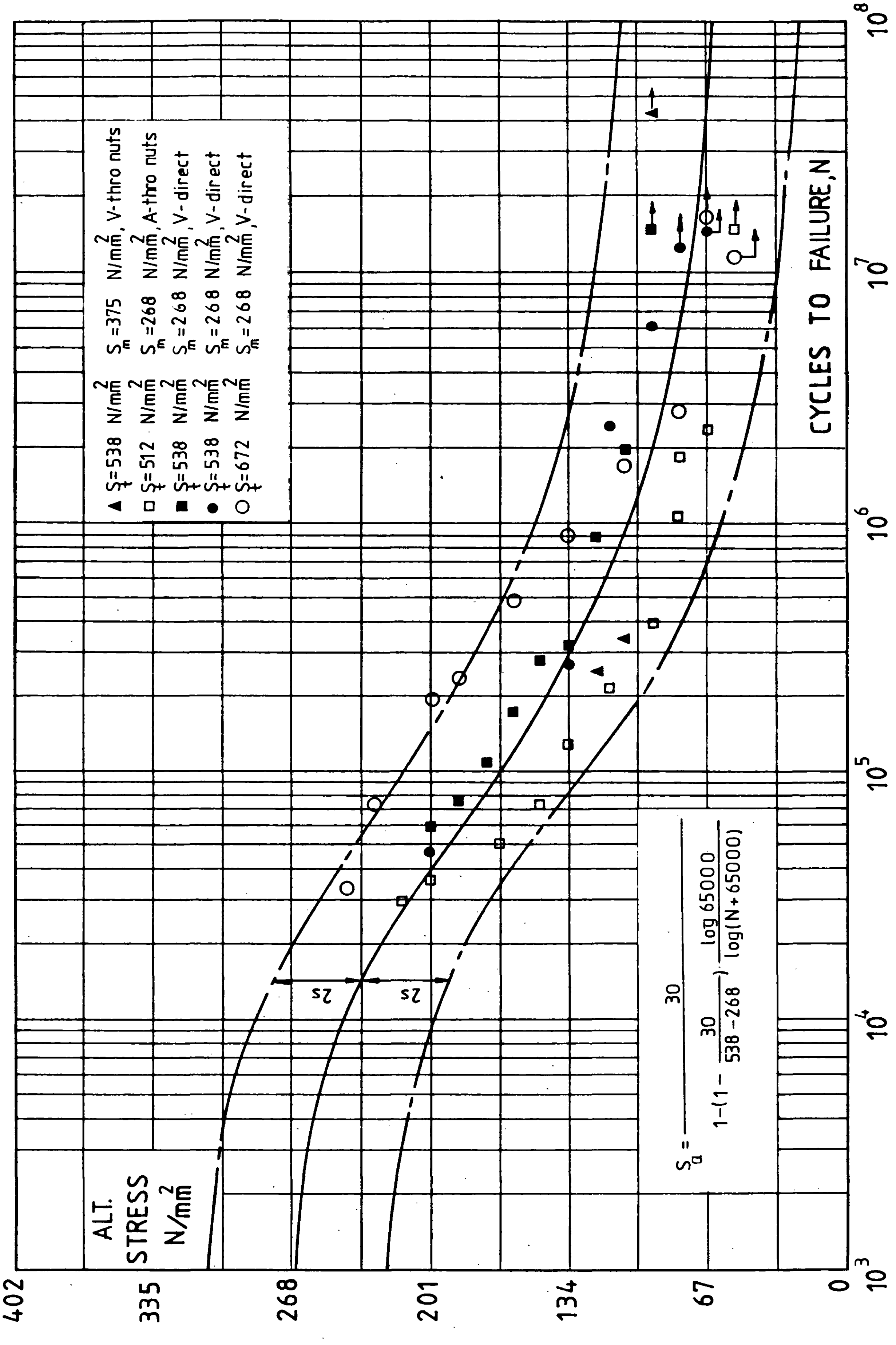


FIG. 5.5 FATIGUE TEST RESULTS OF THE ϕ12 SCREWED BAR AT 20 & 28 KN MEAN LOAD

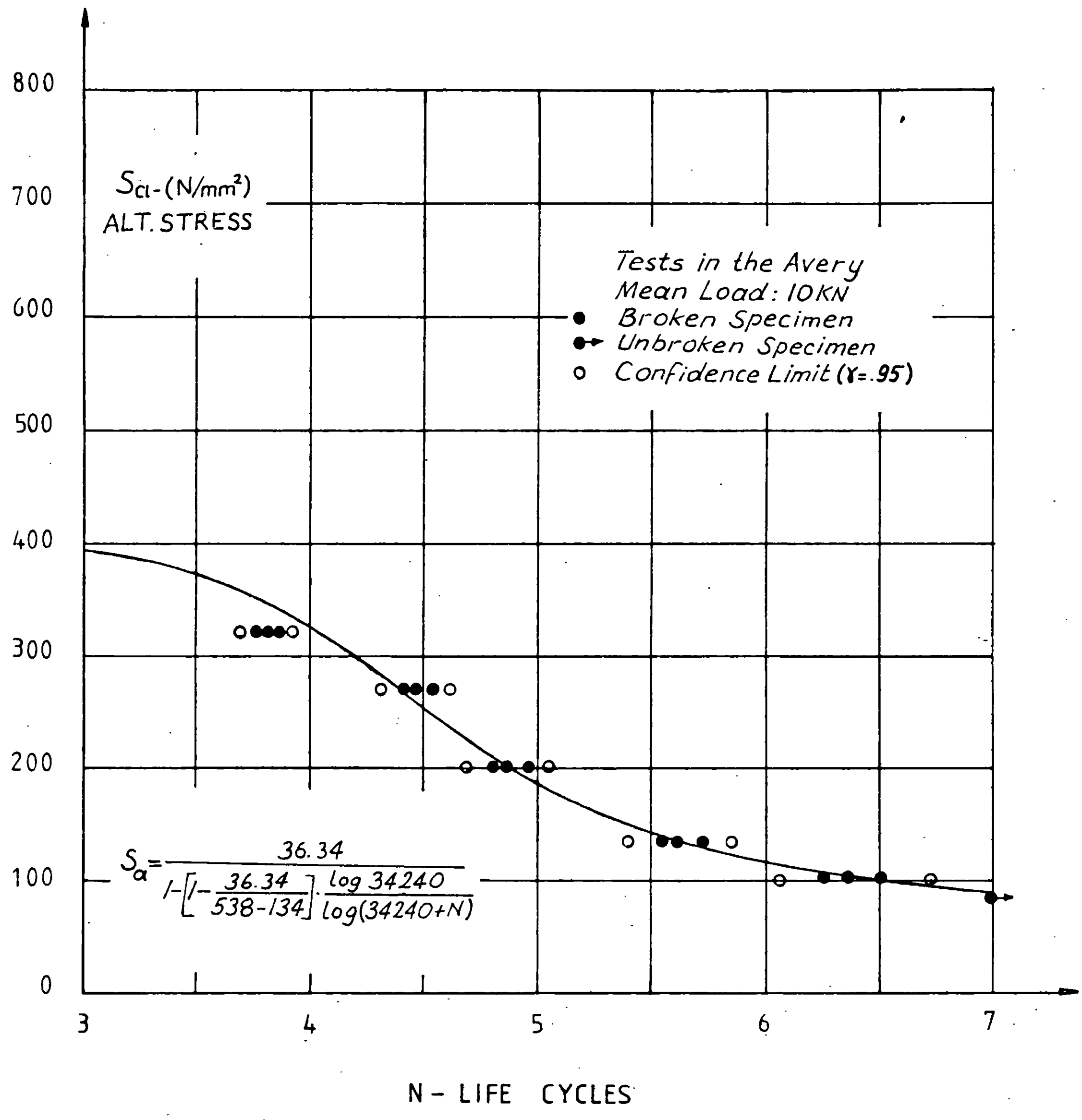


FIG.5.6 FATIGUE LIFE OF THE SCREWED BAR SHOWING CONFIDENCE LIMITS

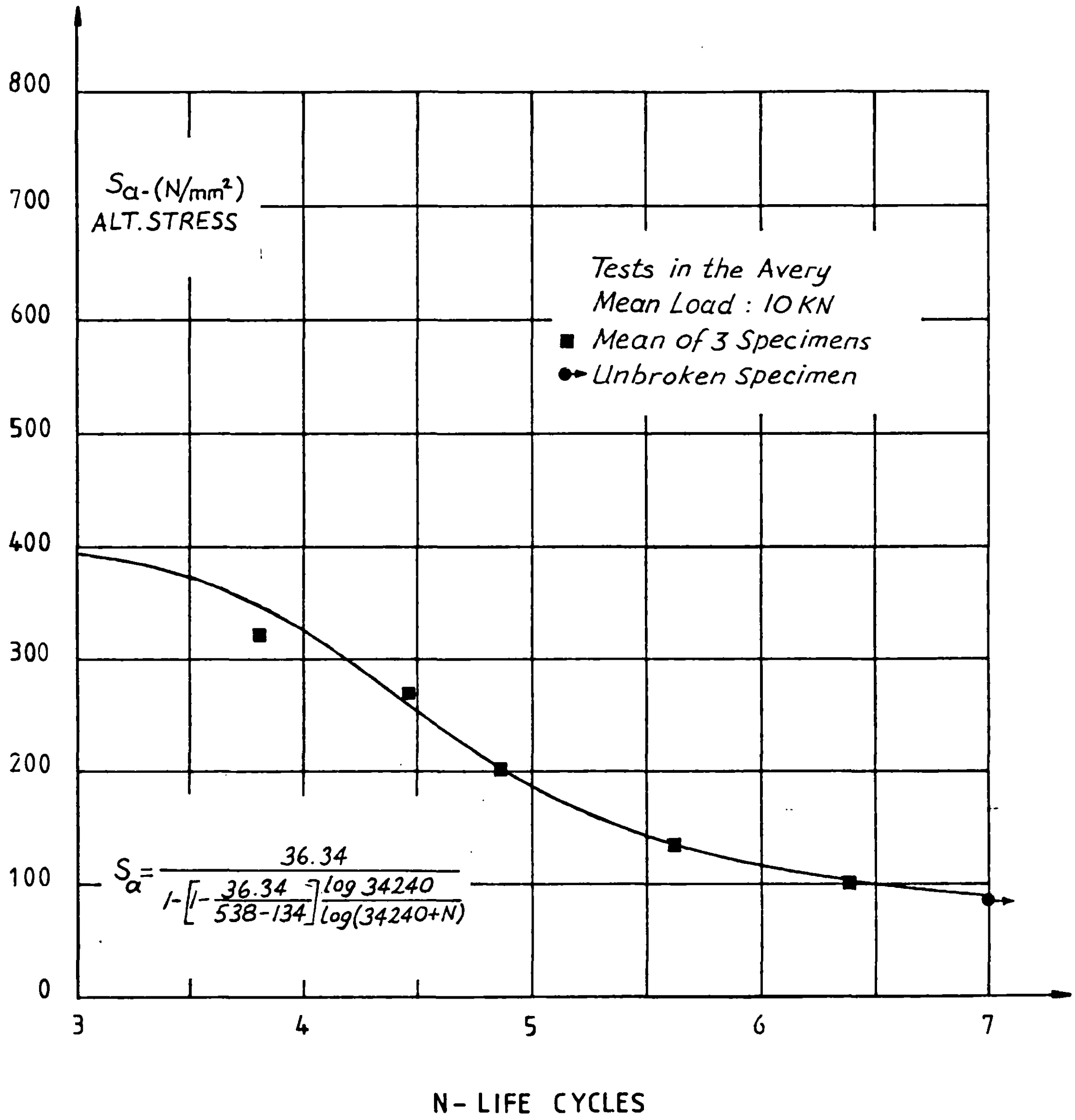


FIG. 5.7 FATIGUE LIFE OF THE SCREWED BAR

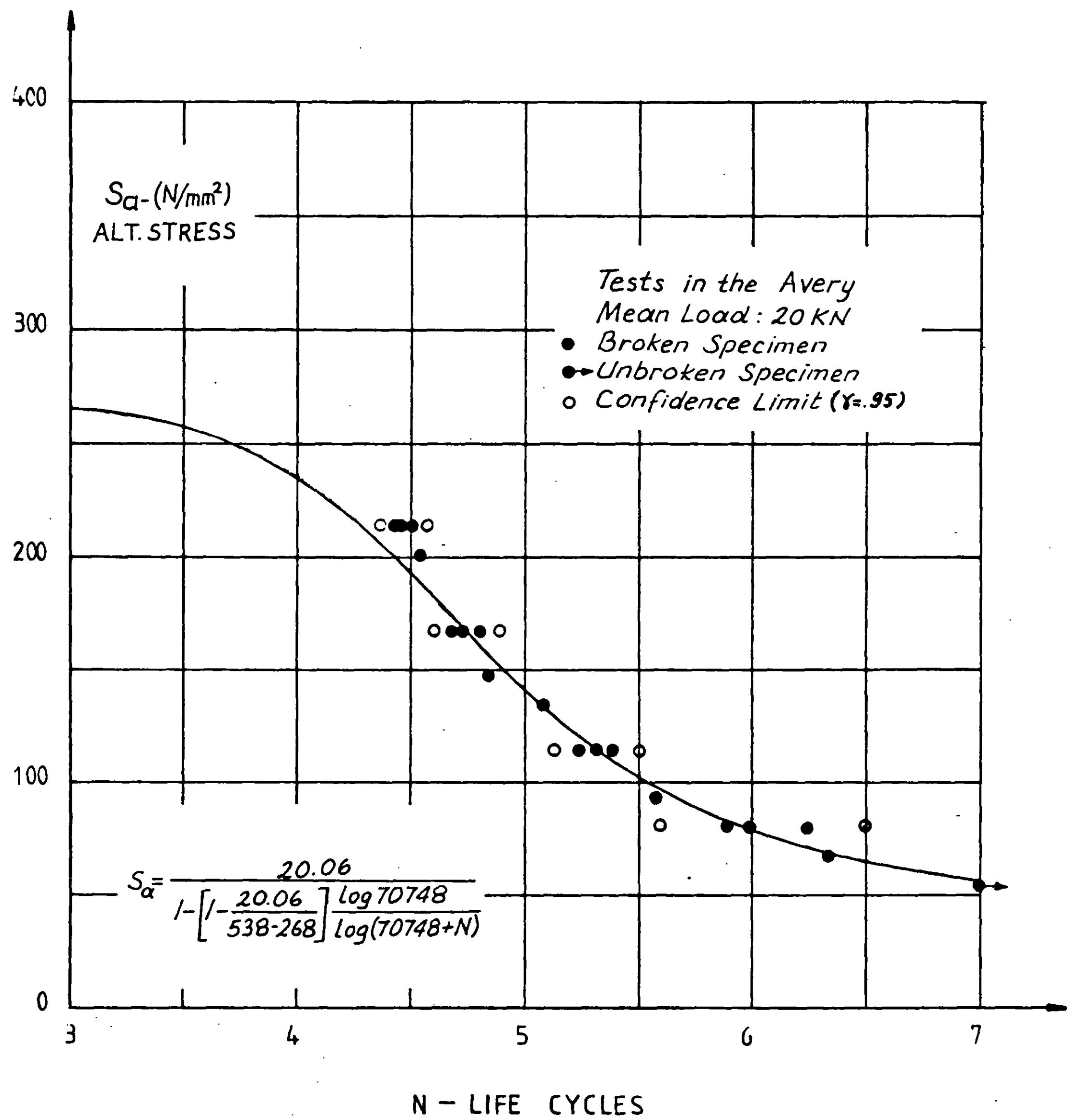


FIG.5.8 FATIGUE LIFE OF THE SCREWED BAR SHOWING CONFIDENCE LIMITS

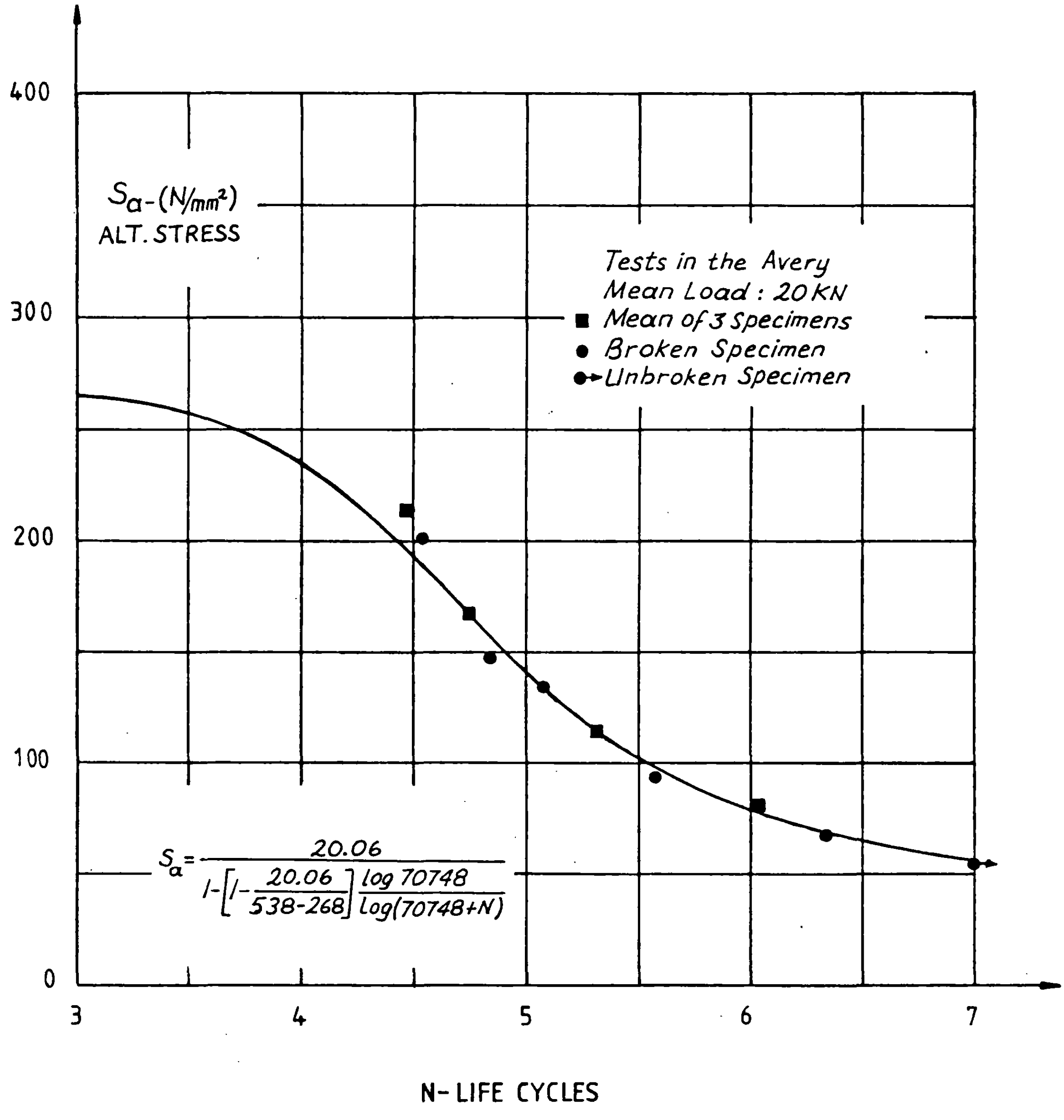


FIG.5_9 FATIGUE LIFE OF THE SCREWED BAR

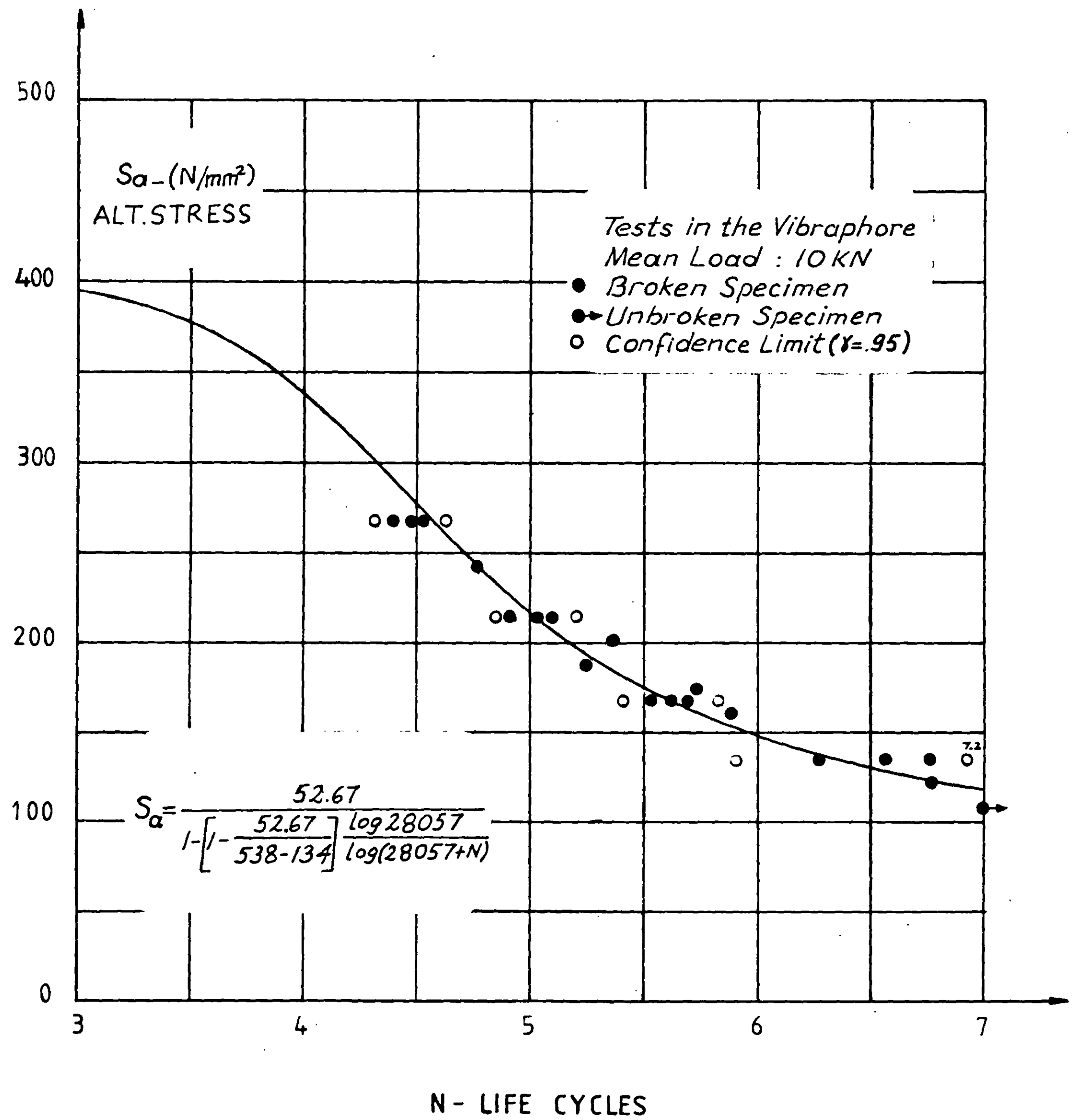


FIG. 5.10 FATIGUE LIFE OF THE SCREWED BAR SHOWING CONFIDENCE LIMITS

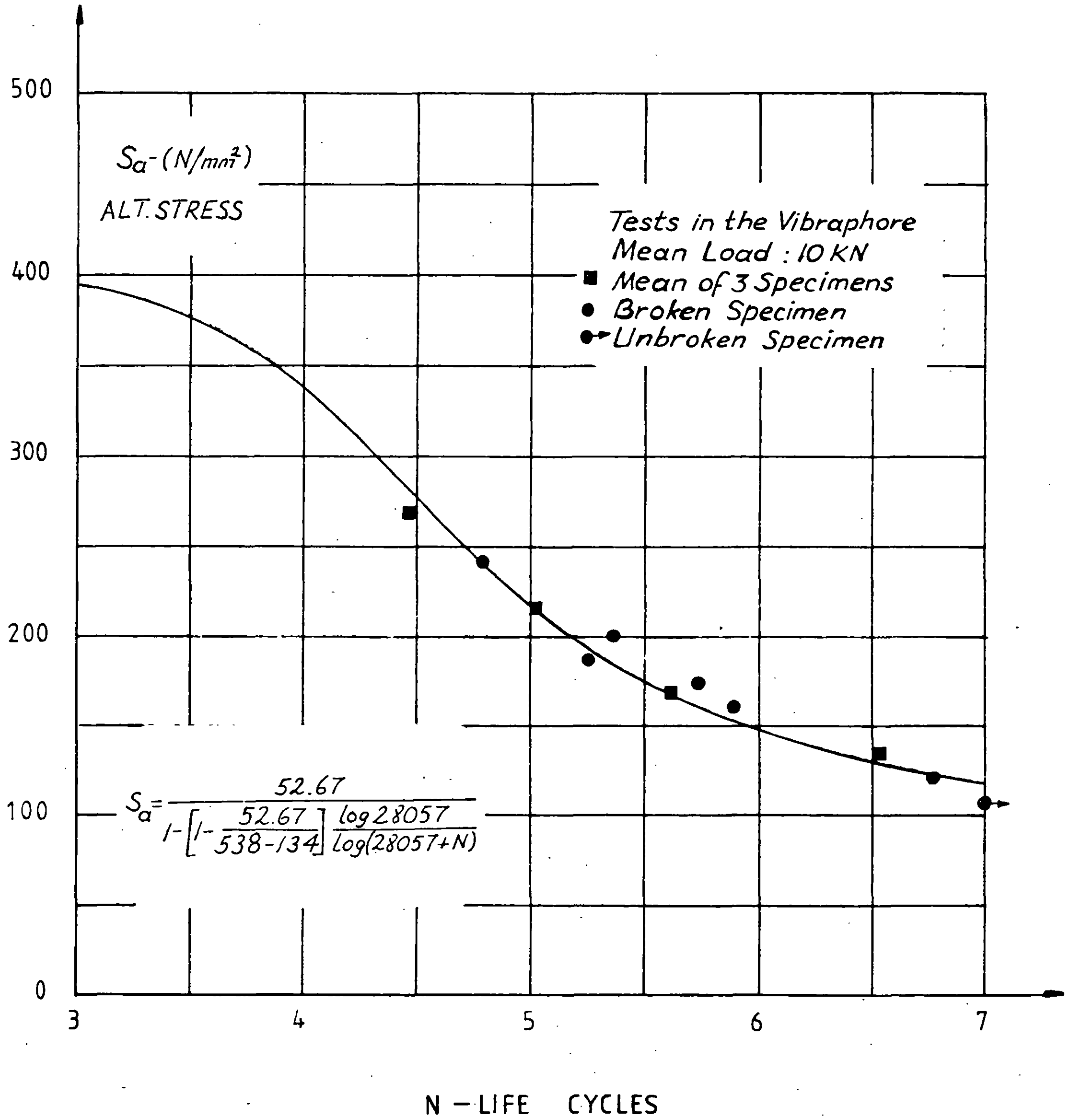


FIG.5.11 FATIGUE LIFE OF THE SCREWED BAR

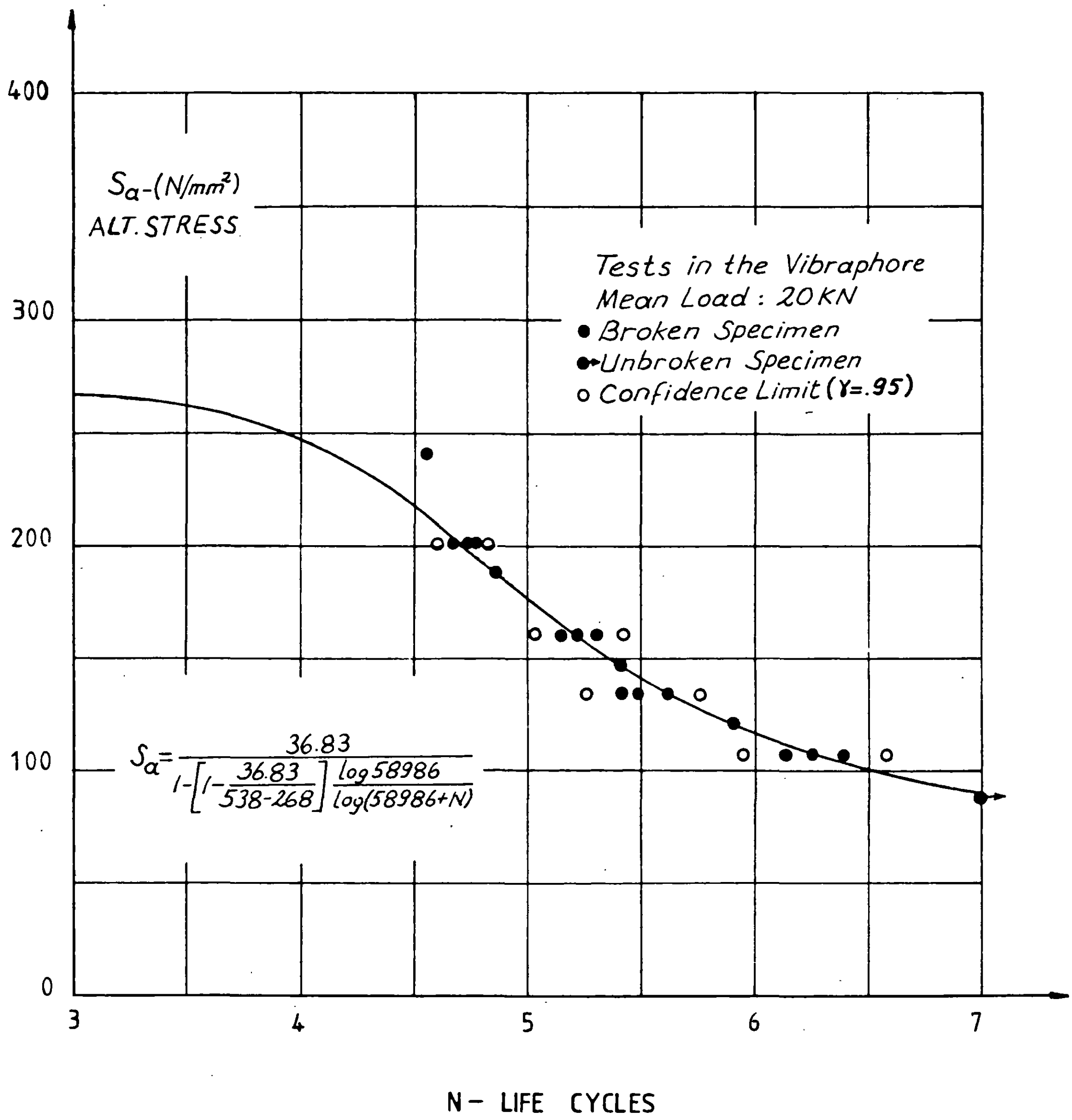


FIG.5.12 FATIGUE LIFE OF THE SCREWED BAR SHOWING CONFIDENCE LIMITS

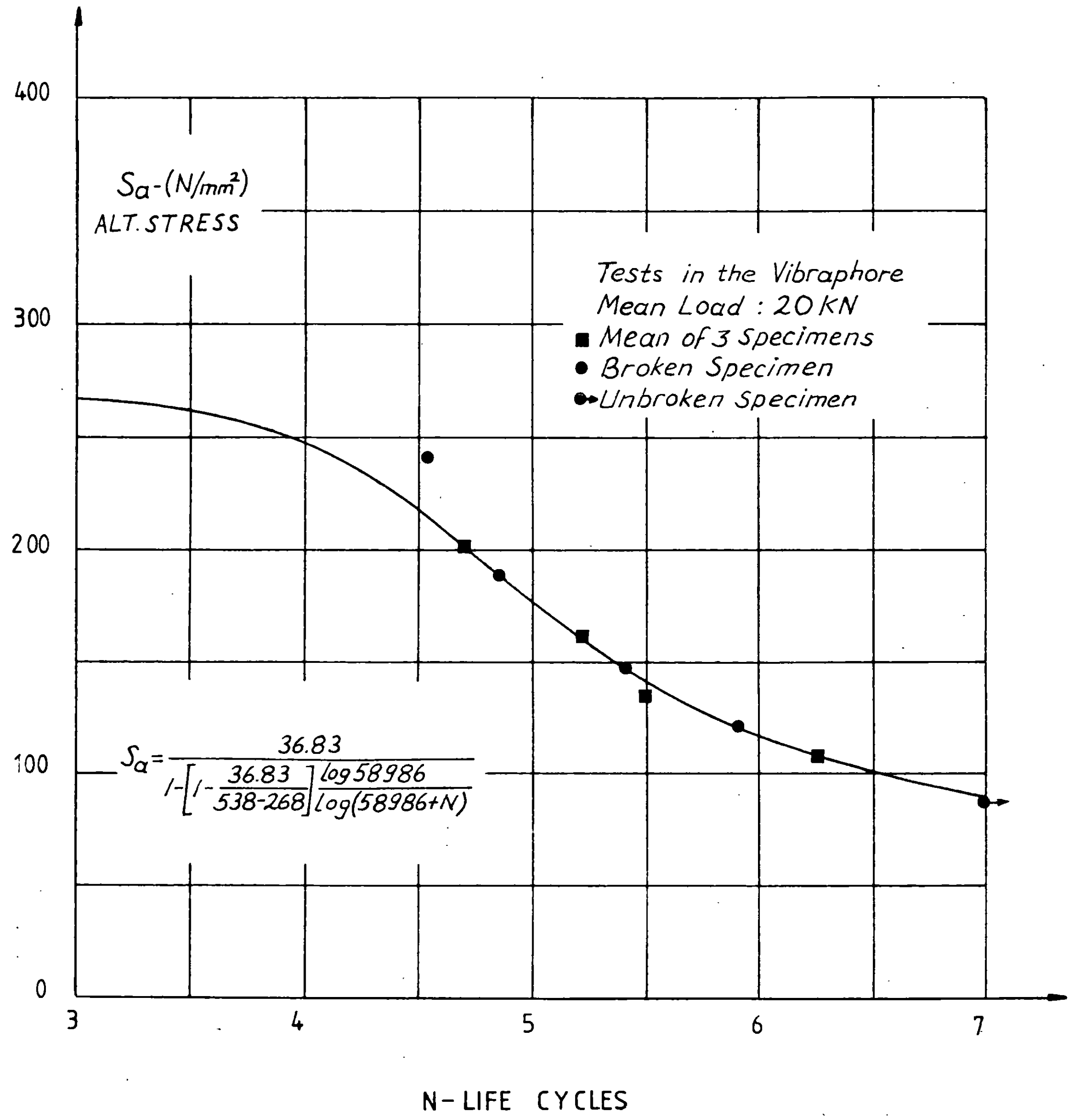


FIG.5_13 FATIGUE LIFE OF THE SCREWED BAR

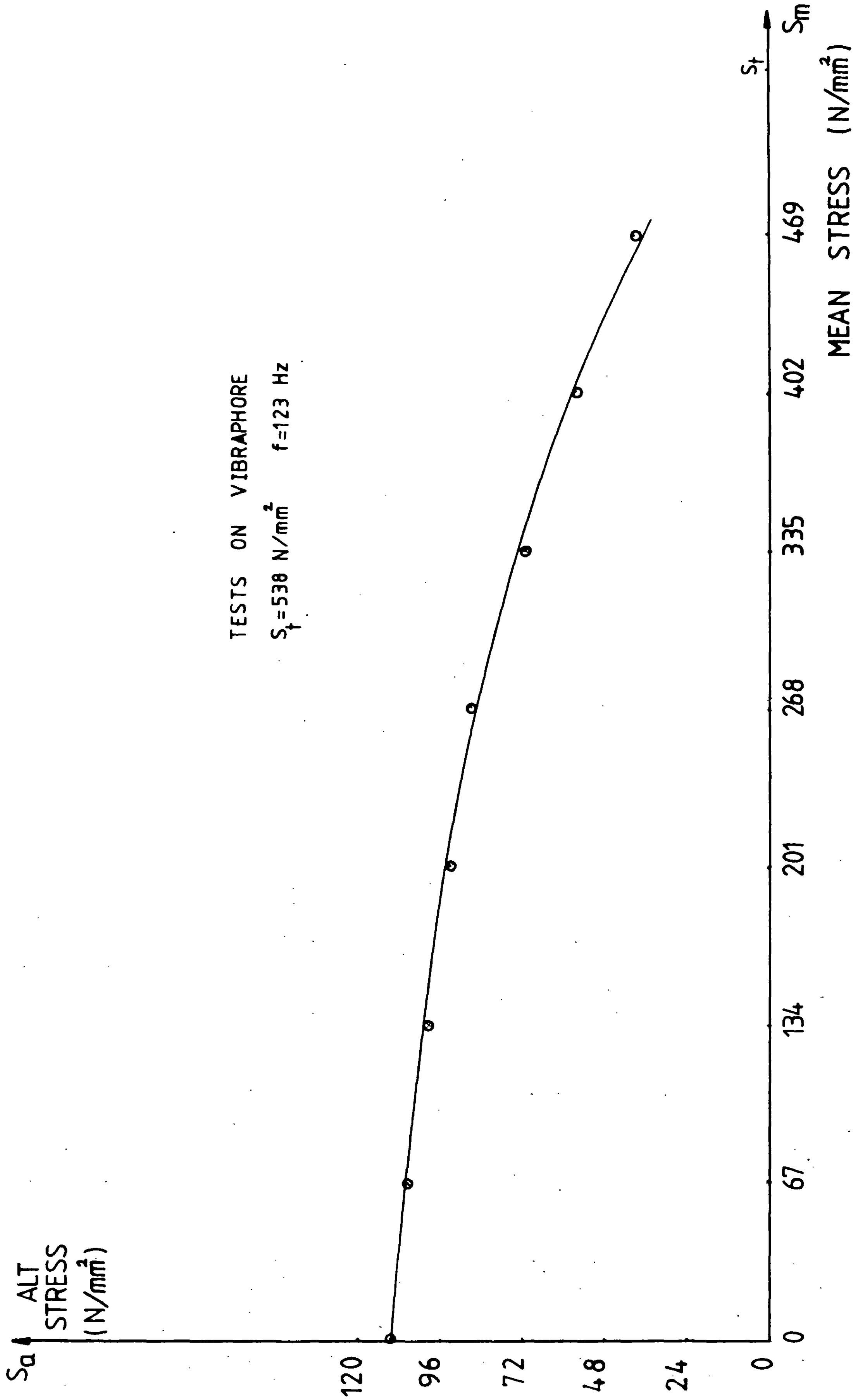


FIG.5.14 THE ENDURANCE STRESS OF Ø12 SCREWED BARS

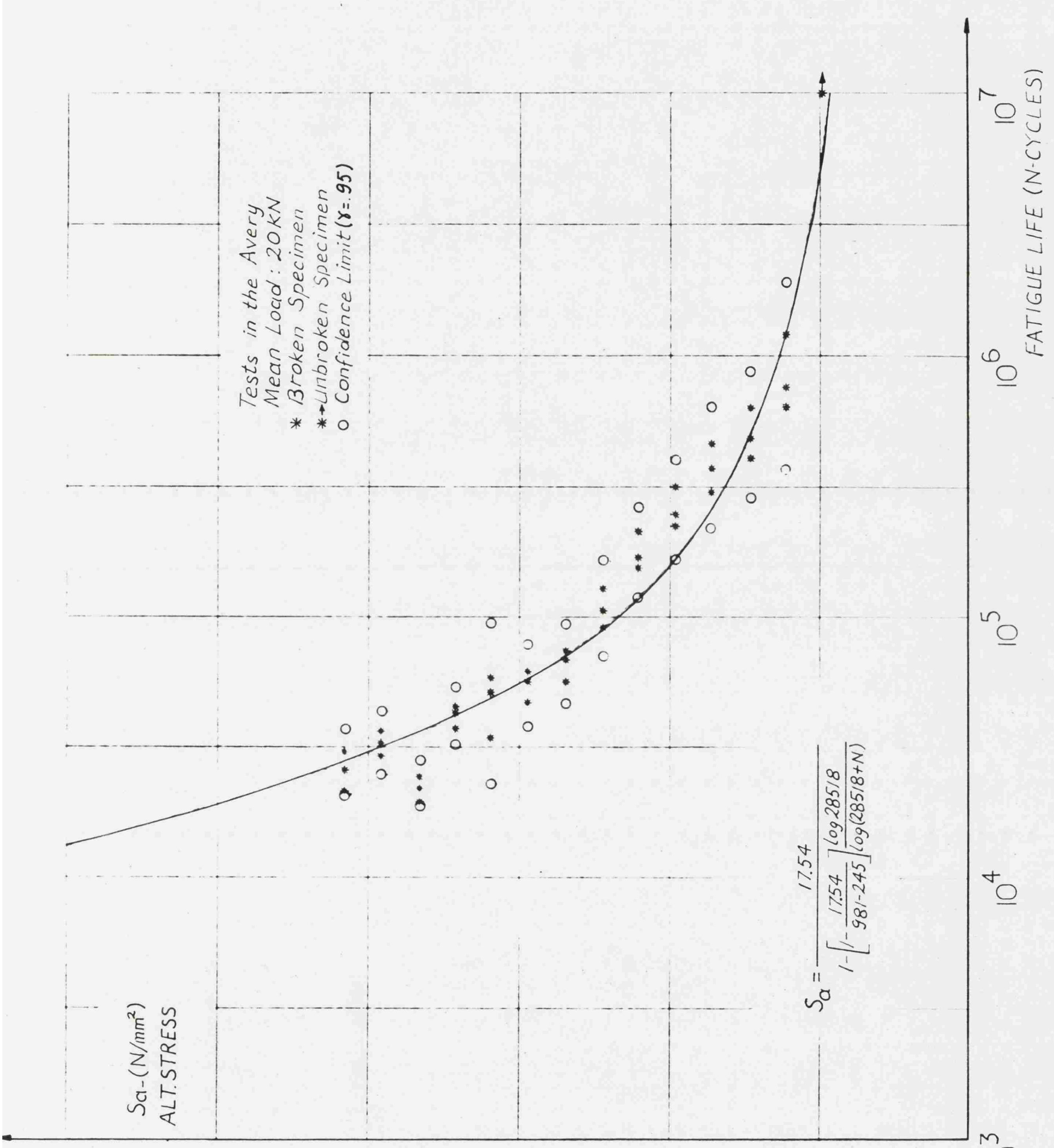


FIG. 6.1 FATIGUE LIFE OF THE Ø12-BOLT SHOWING CONFIDENCE LIMITS

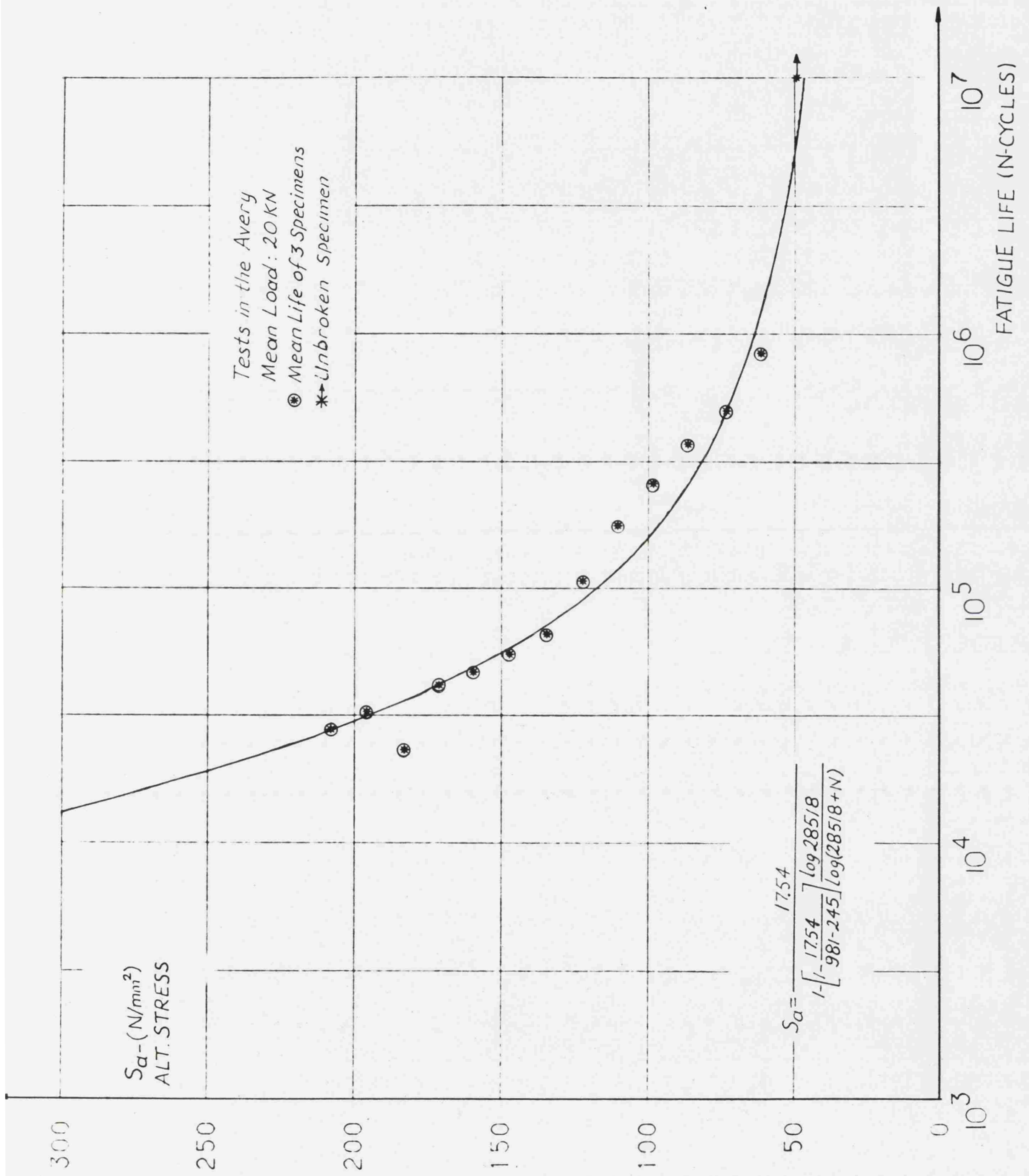


FIG.6.2 FATIGUE LIFE OF THE Ø12-BOLT

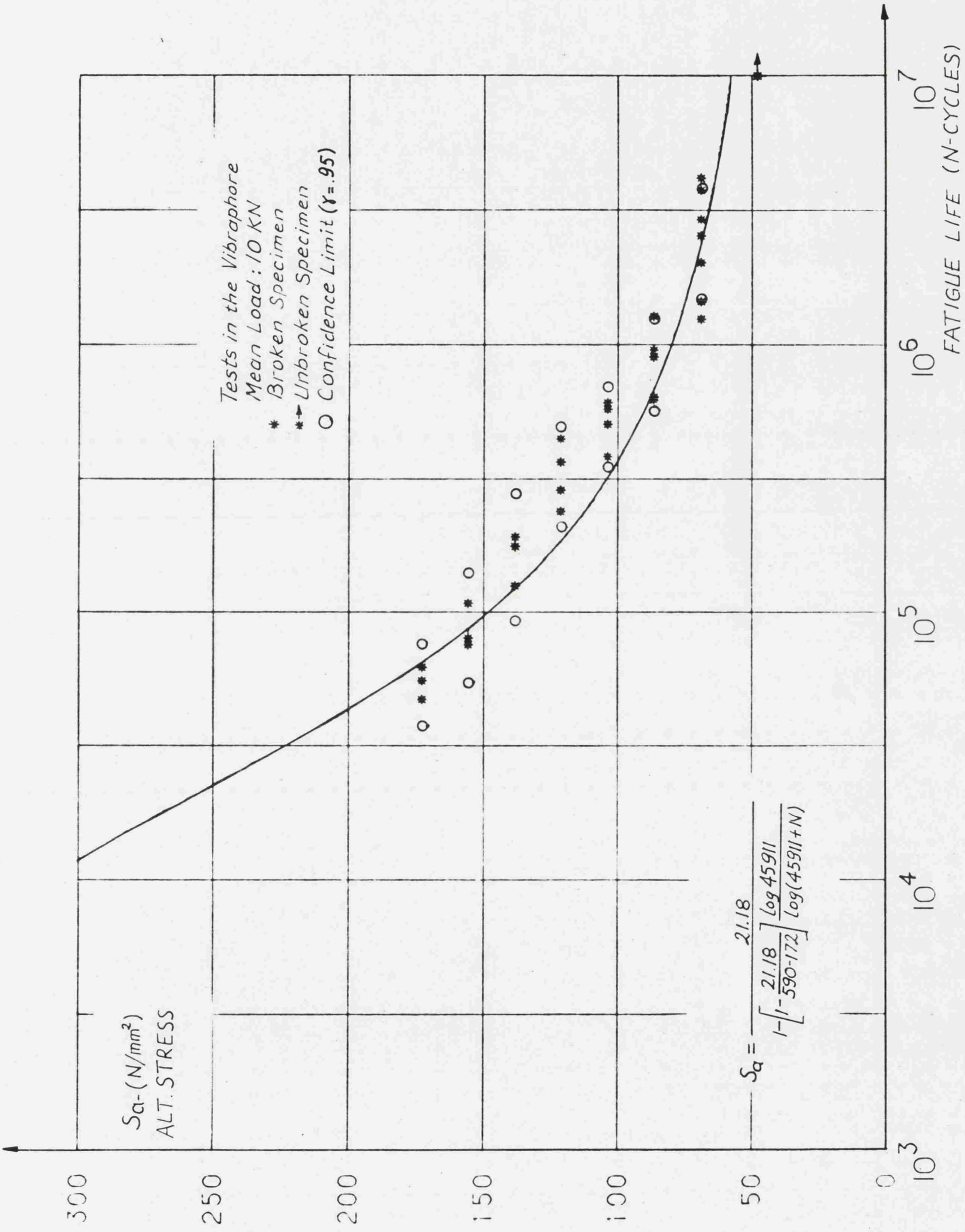


FIG. 6.3 FATIGUE LIFE OF THE Ø10-BOLT SHOWING CONFIDENCE LIMITS

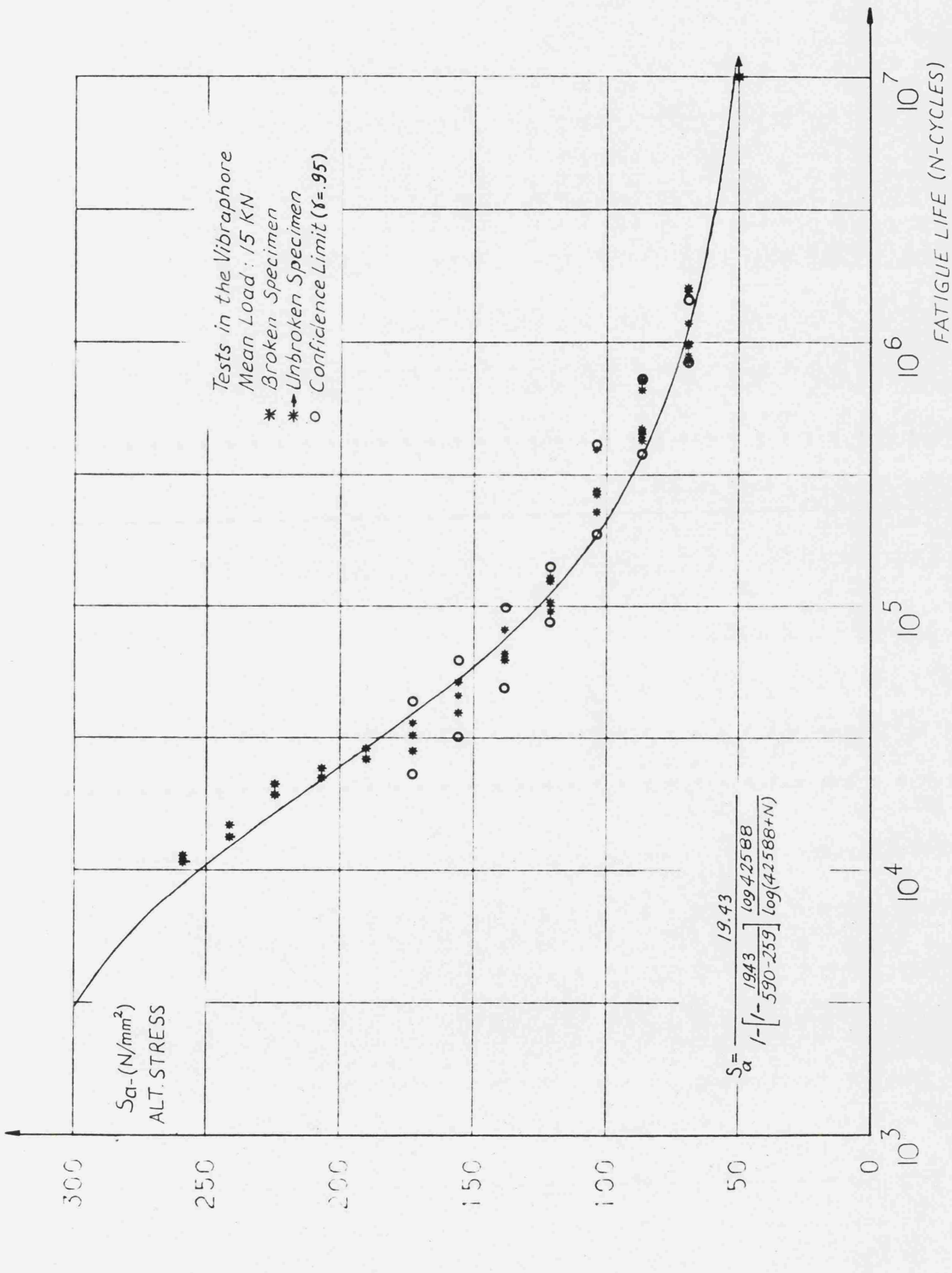


FIG.6.4 FATIGUE LIFE OF THE Ø10-BOLT SHOWING CONFIDENCE LIMITS

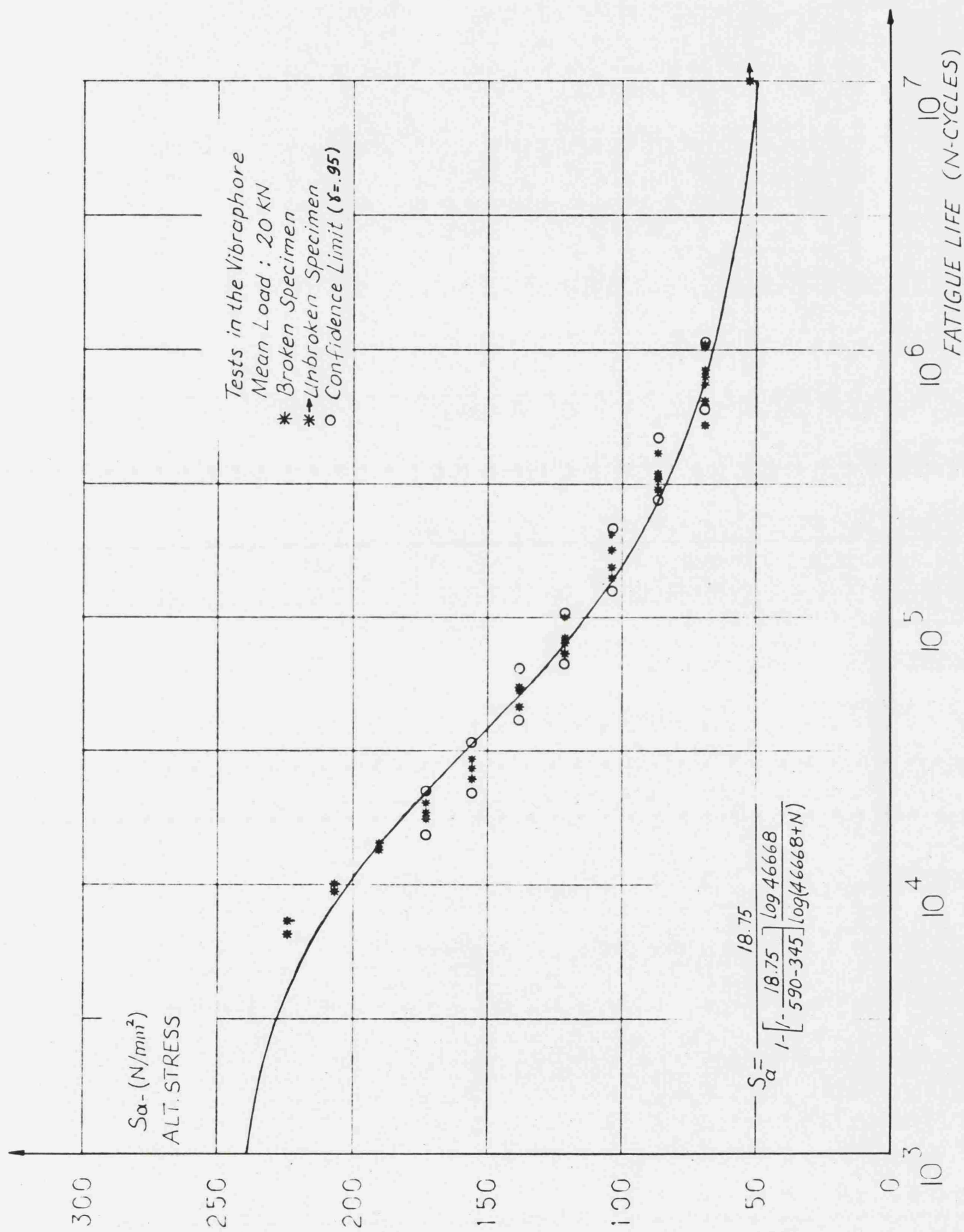


FIG.6.5 FATIGUE LIFE OF THE Ø10-BOLT SHOWING CONFIDENCE LIMITS

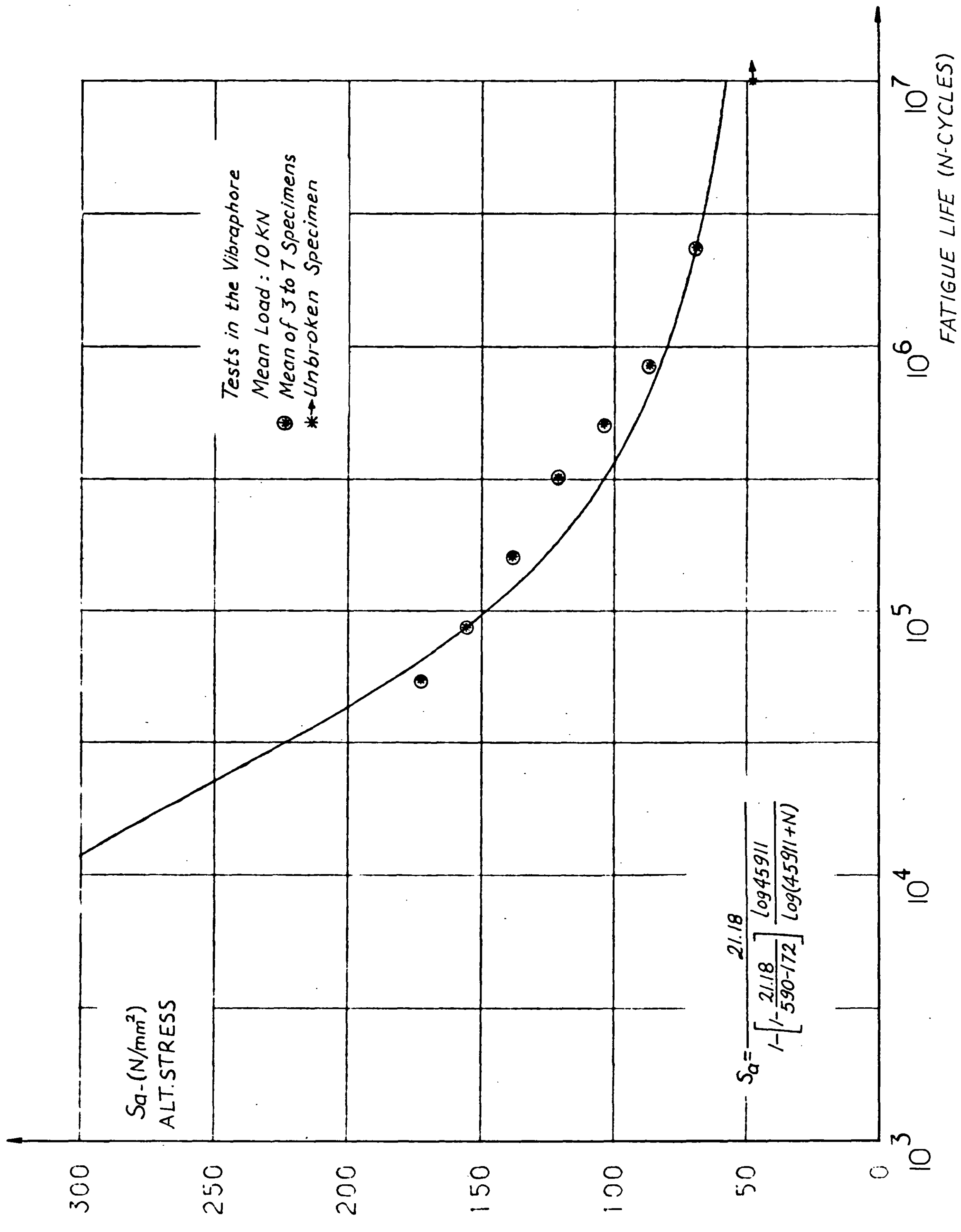


FIG. 6.6 FATIGUE LIFE OF THE Φ10-BOLT

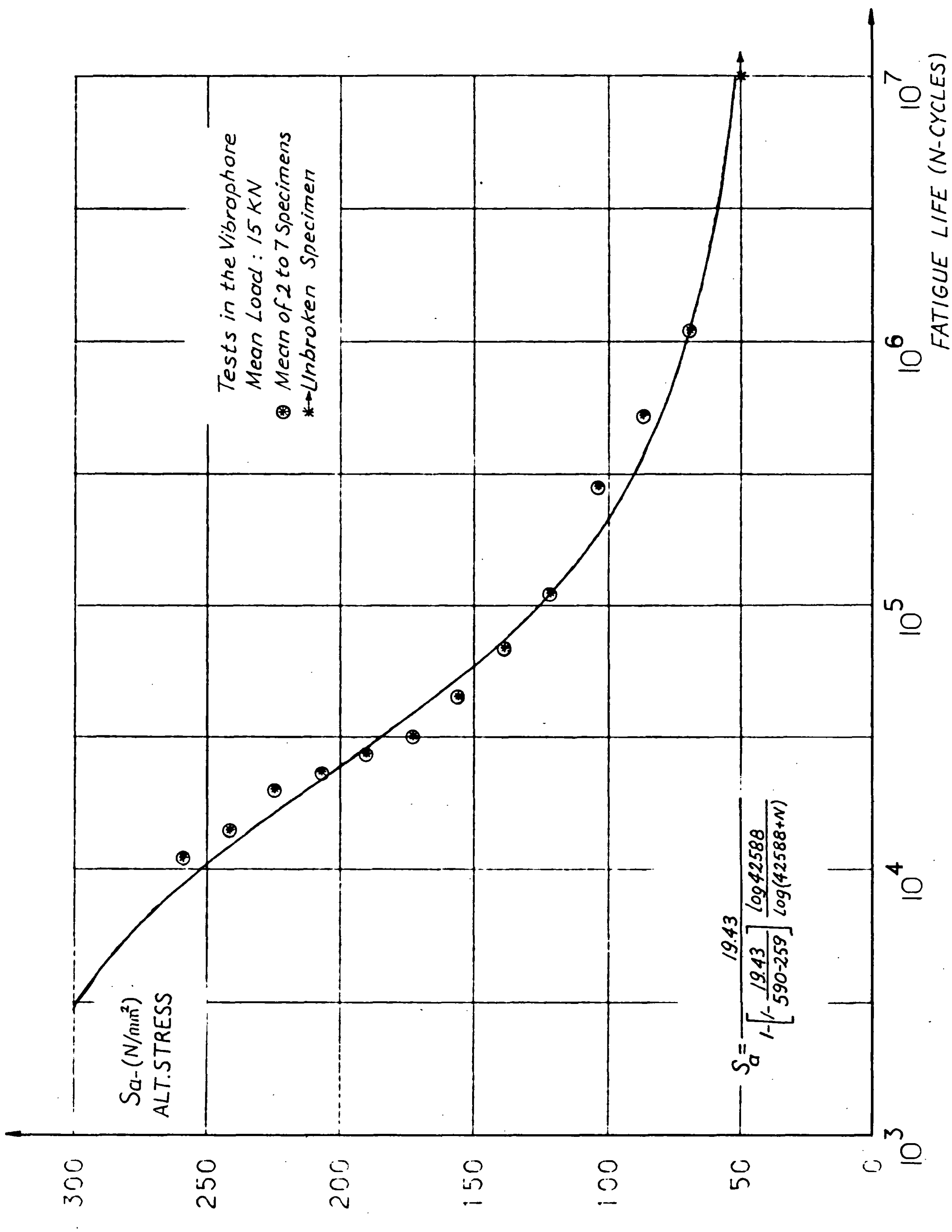


FIG. 6.7 FATIGUE LIFE OF THE Ø10-BOLT

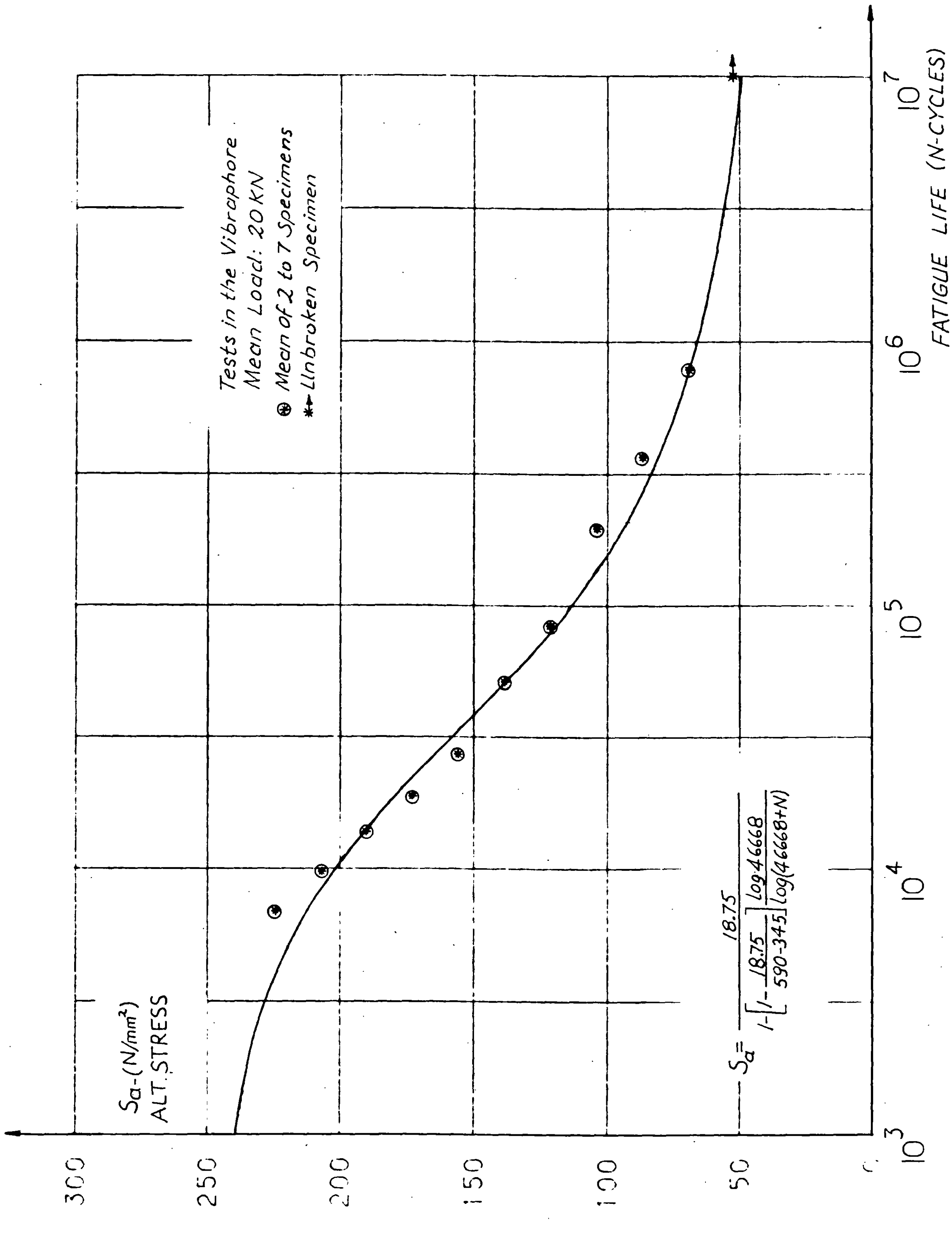


FIG. 6.8 FATIGUE LIFE OF THE Ø10-BOLT

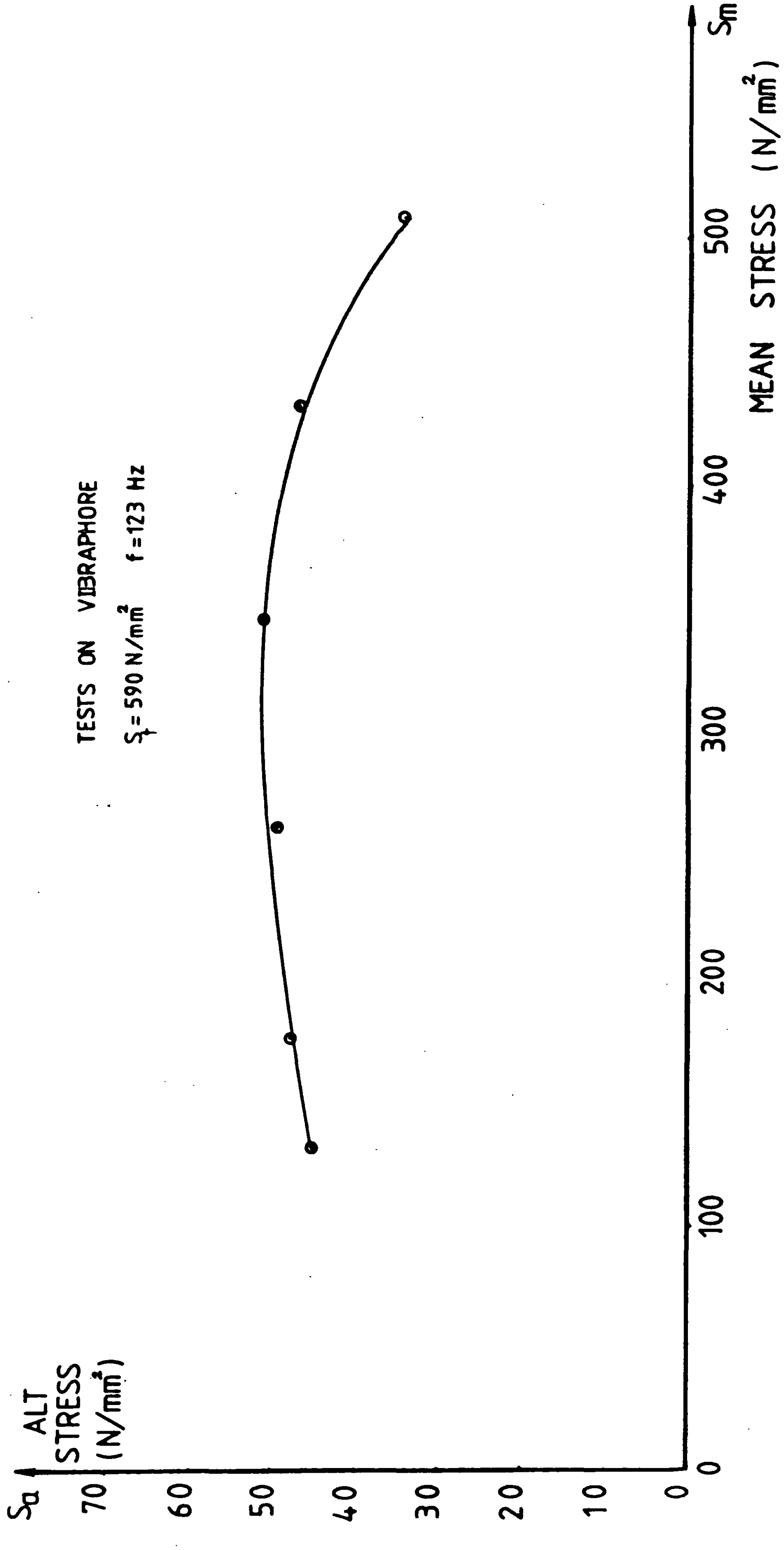


FIG. 6.9 THE ENDURANCE STRESS OF Ø10 BOLTS

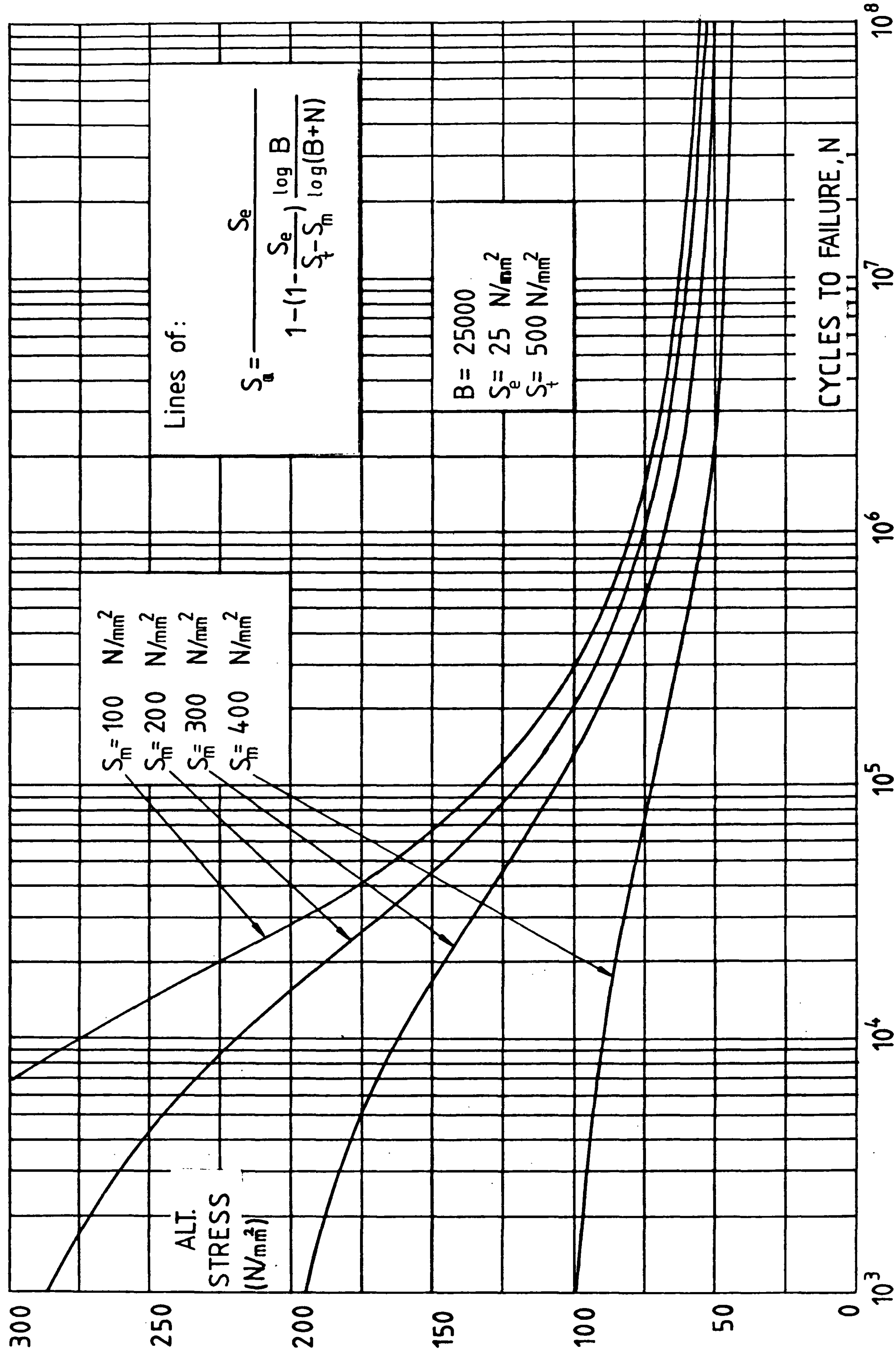


FIG.8.1 EFFECT OF MEAN STRESS UPON JEFFERSON'S CURVE

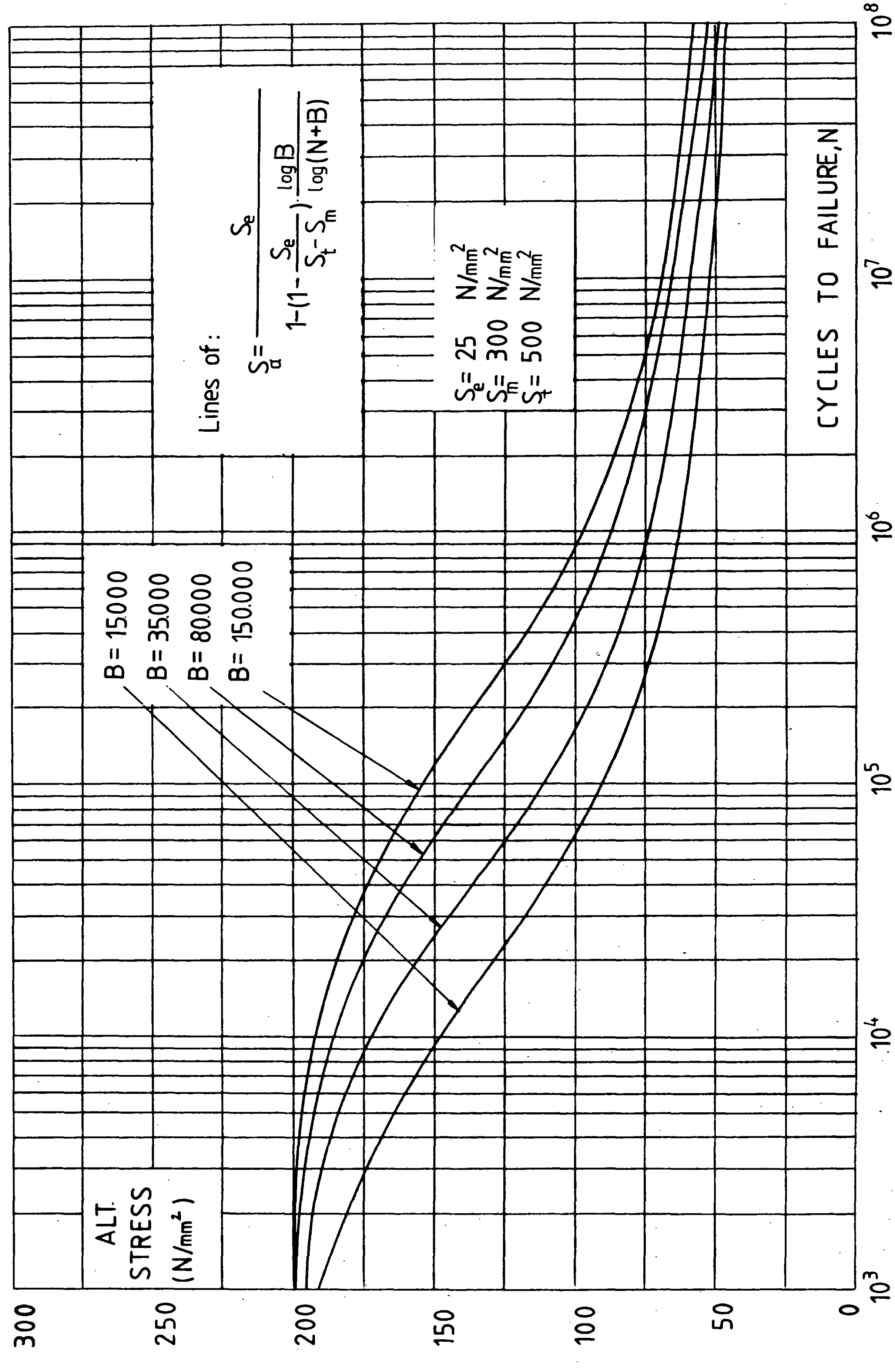


FIG. 8.2 EFFECT OF EQUATION CONSTANT, B, UPON JEFFERSON'S CURVE

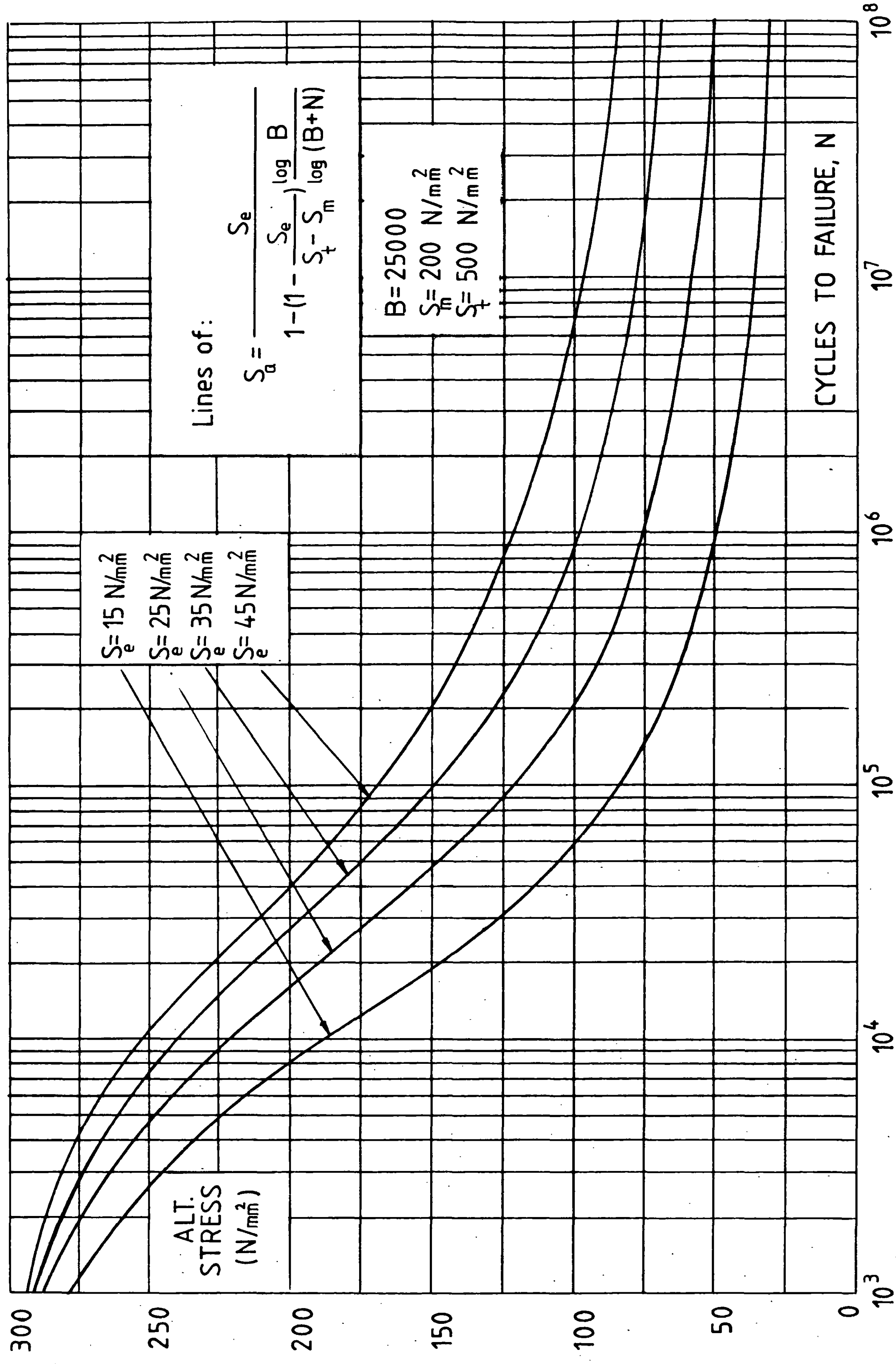


FIG. 8.3 EFFECT OF ENDURANCE STRESS UPON JEFFERSON'S CURVE

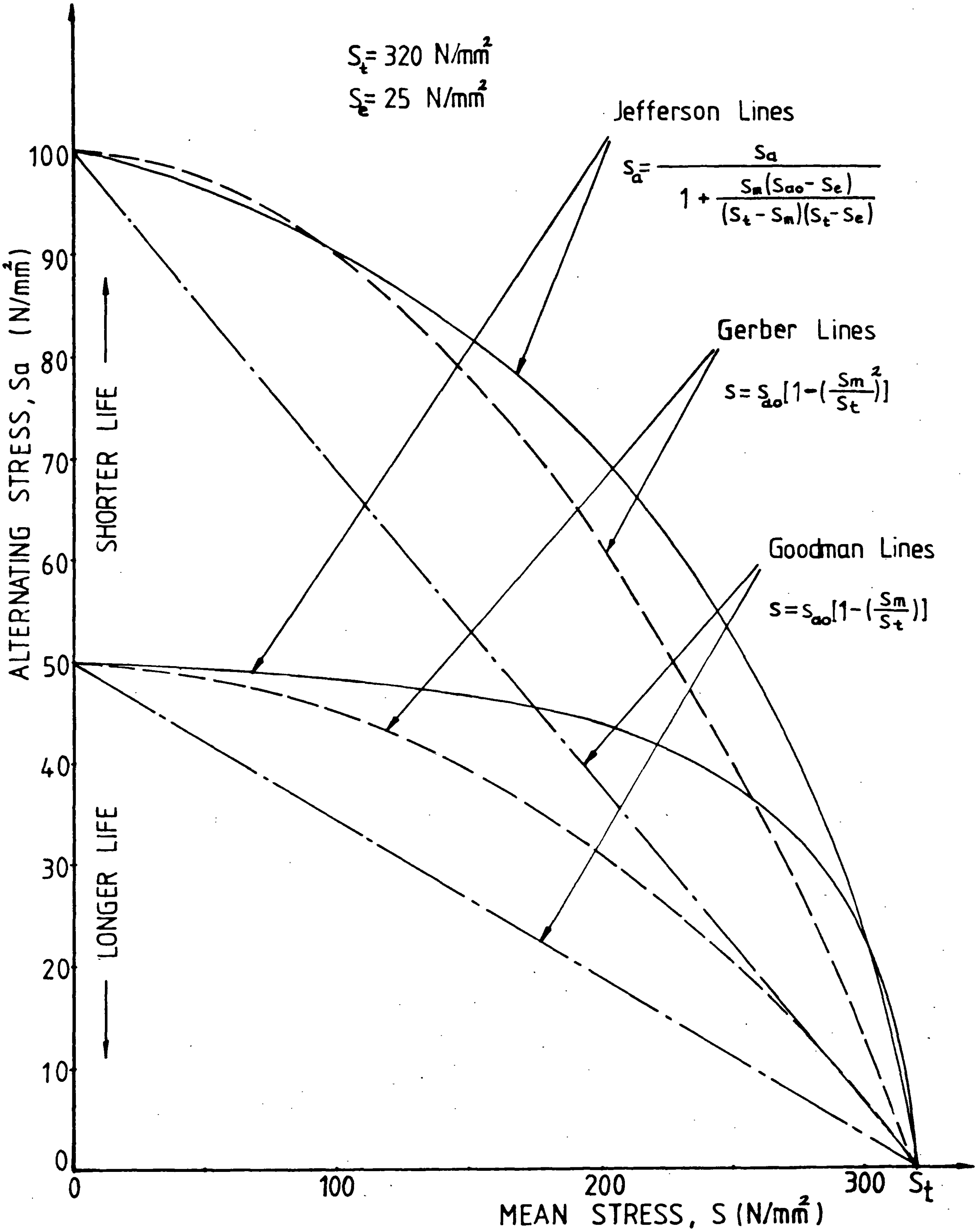


FIG 8.4 EFFECT OF ALTERNATING STRESS UPON MEAN STRESS DISTRIBUTIONS

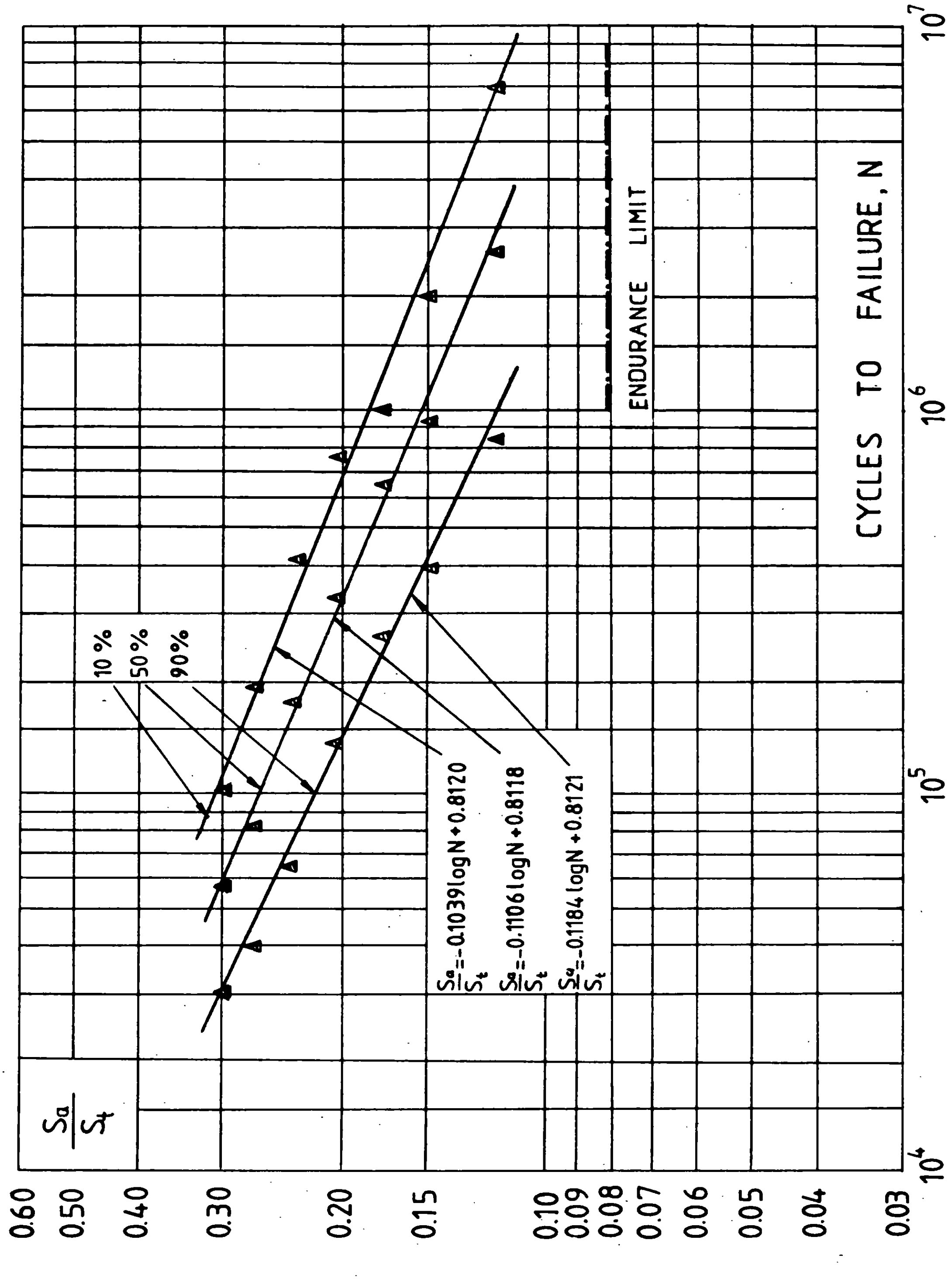


FIG.9.3a SURVIVAL CURVES OF THE ϕ10 BOLT WITH A CONFIDENCE LEVEL $\gamma = 0.90$ AT 10 KN MEAN LOAD

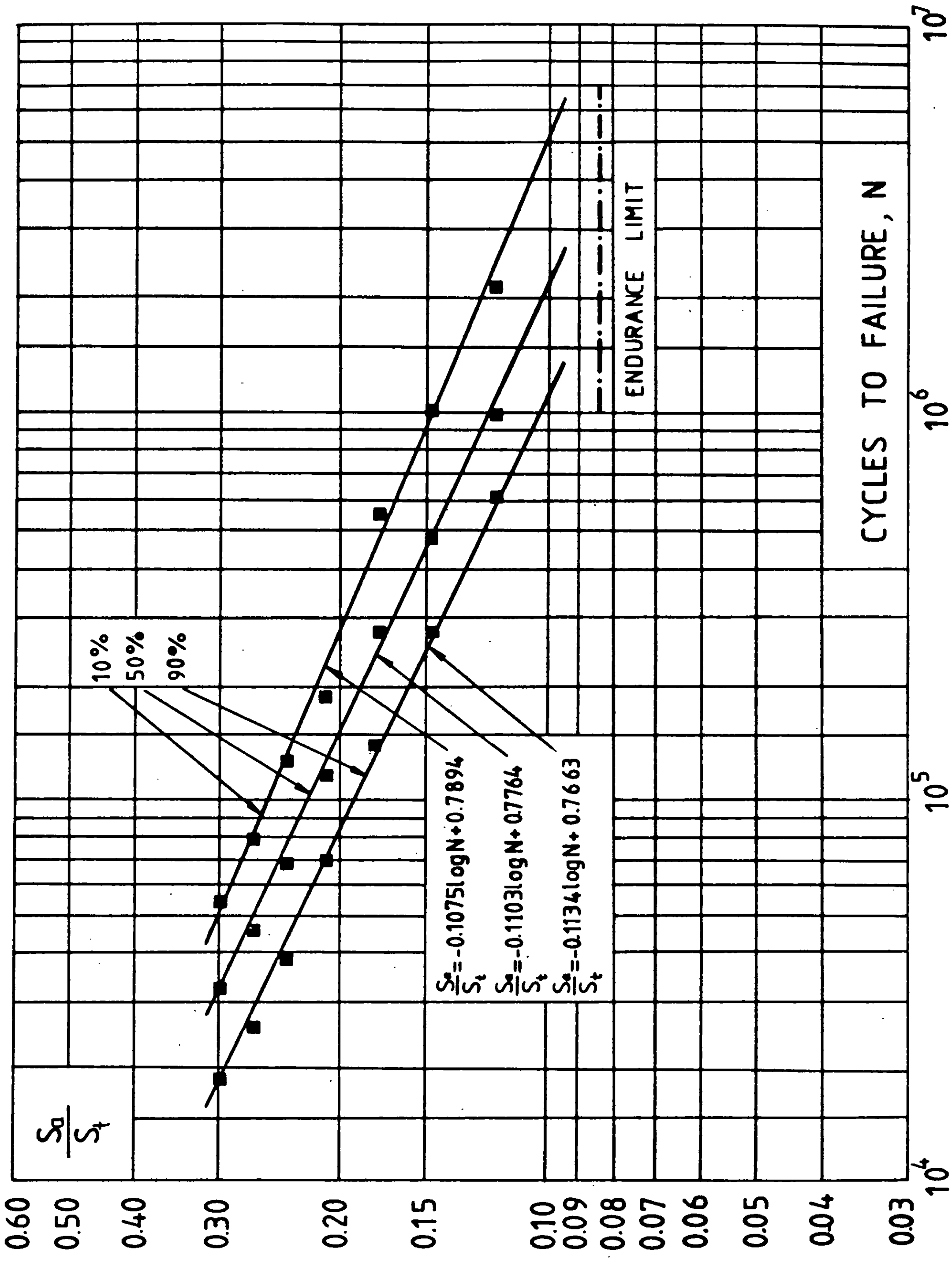


FIG.9.3b SURVIVAL CURVES OF THE Ø10-BOLT WITH A CONFIDENCE LEVEL $\gamma = 0.90$ AT 15 KN MEAN LOAD

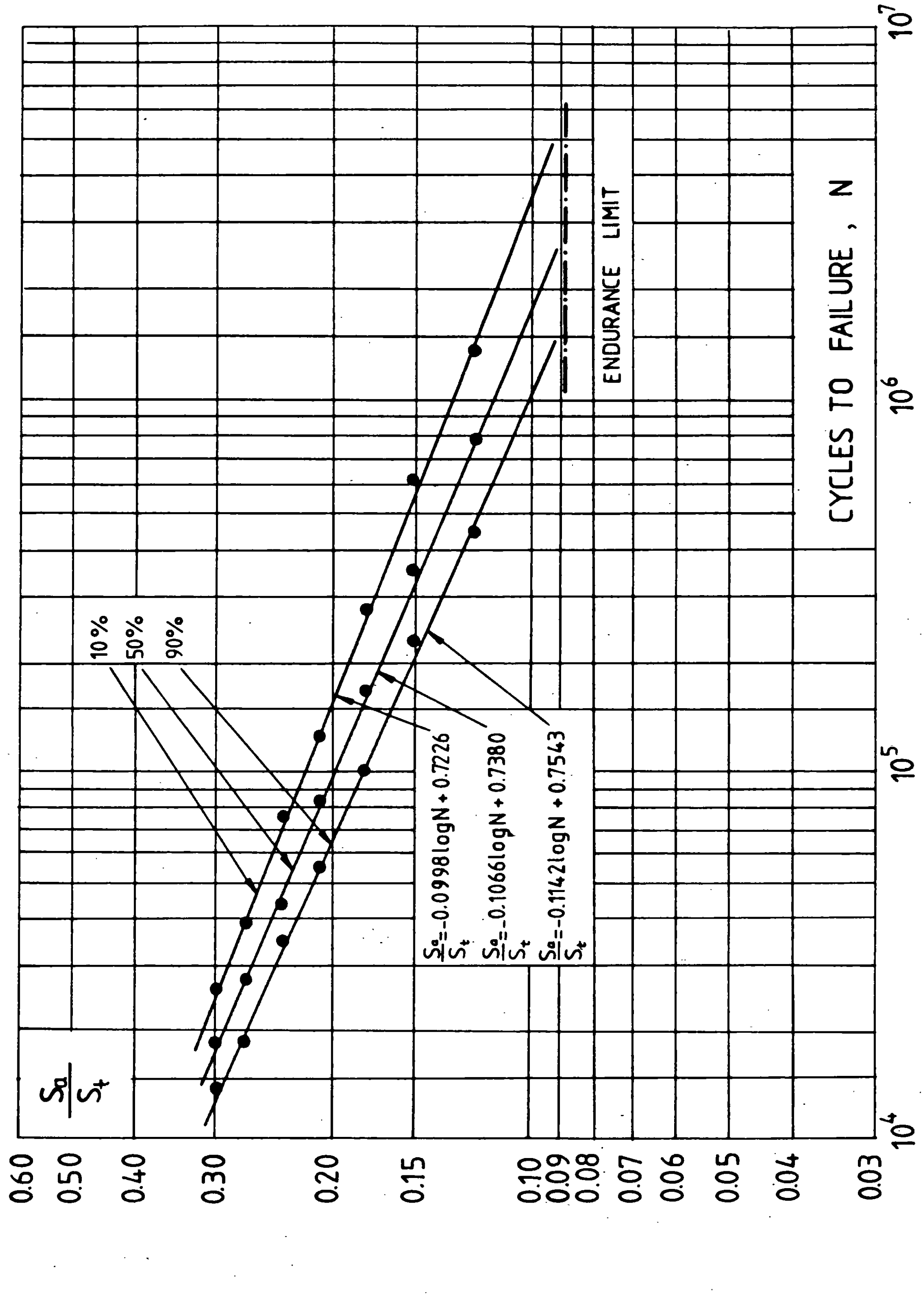


FIG.9.3c SURVIVAL CURVES OF THE ϕ10 BOLT WITH A CONFIDENCE LEVEL $\gamma = 0.90$ AT 20 KN MEAN LOAD

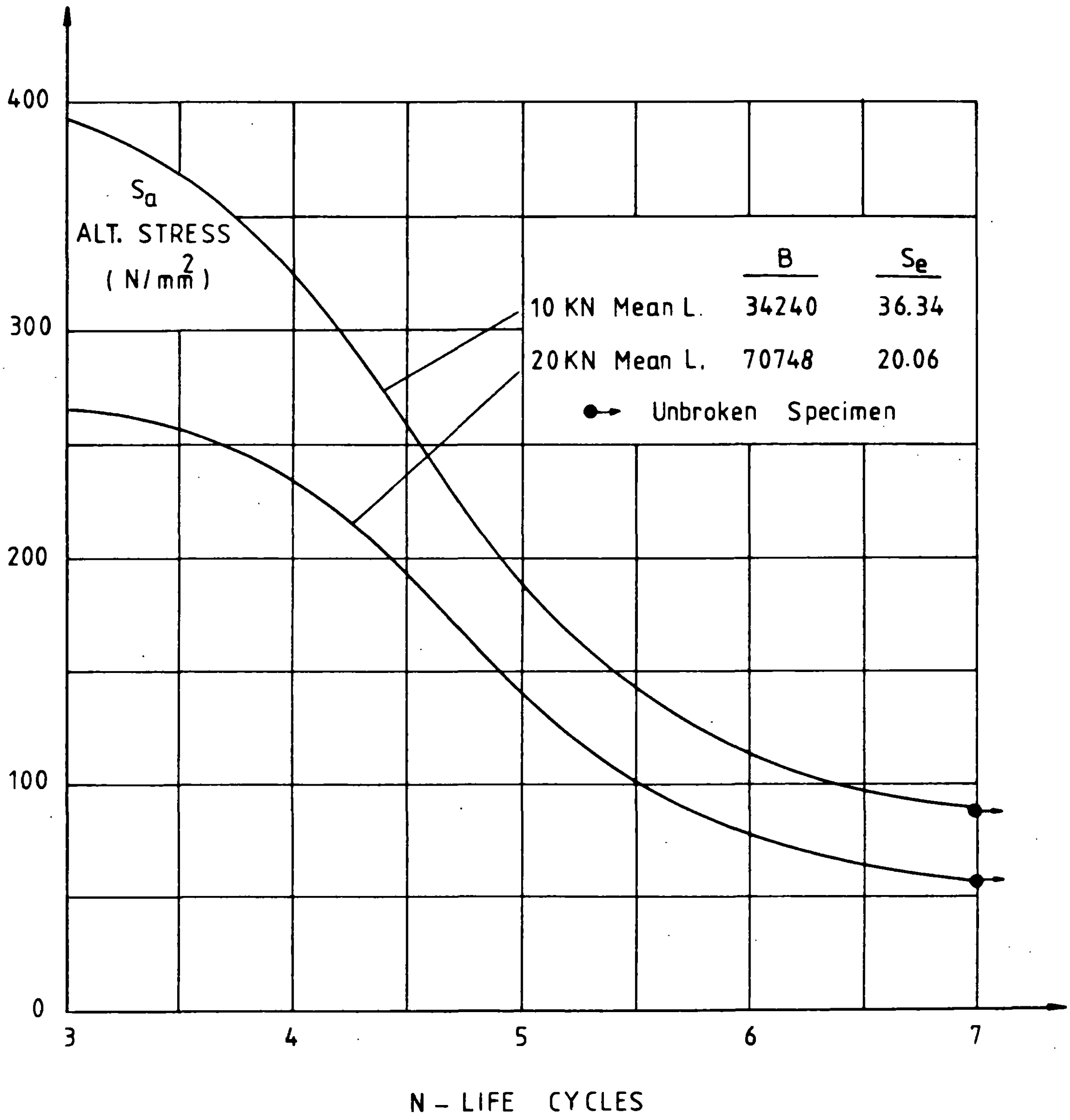


FIG.10.1 THEORETICAL MEAN LIFE OF THE SCREWED BAR (AVERY)

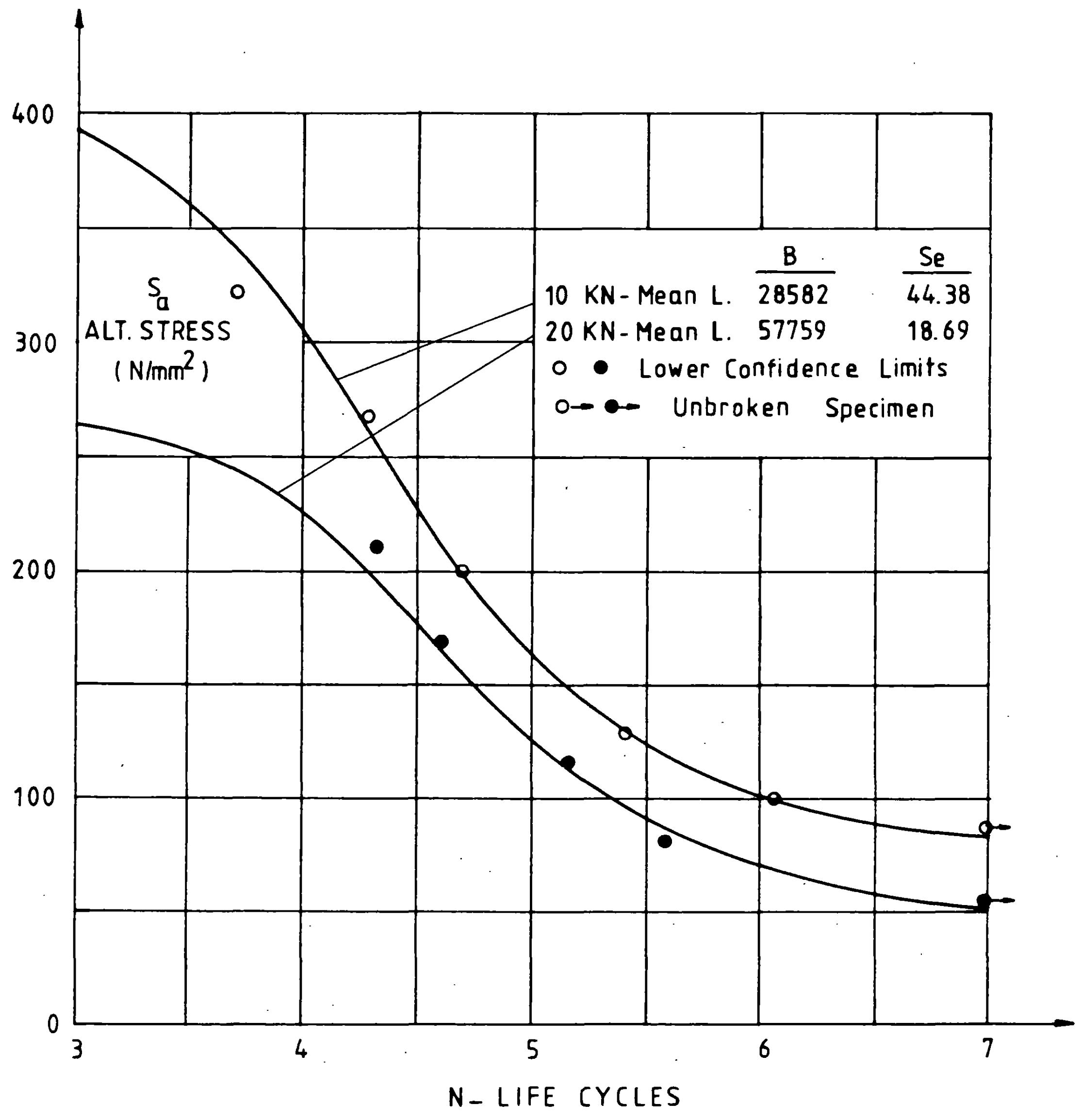


FIG.10_2 THEORETICAL LIFE OF THE SCREWED BAR AT 95% CONFIDENCE LEVEL (AVERY)

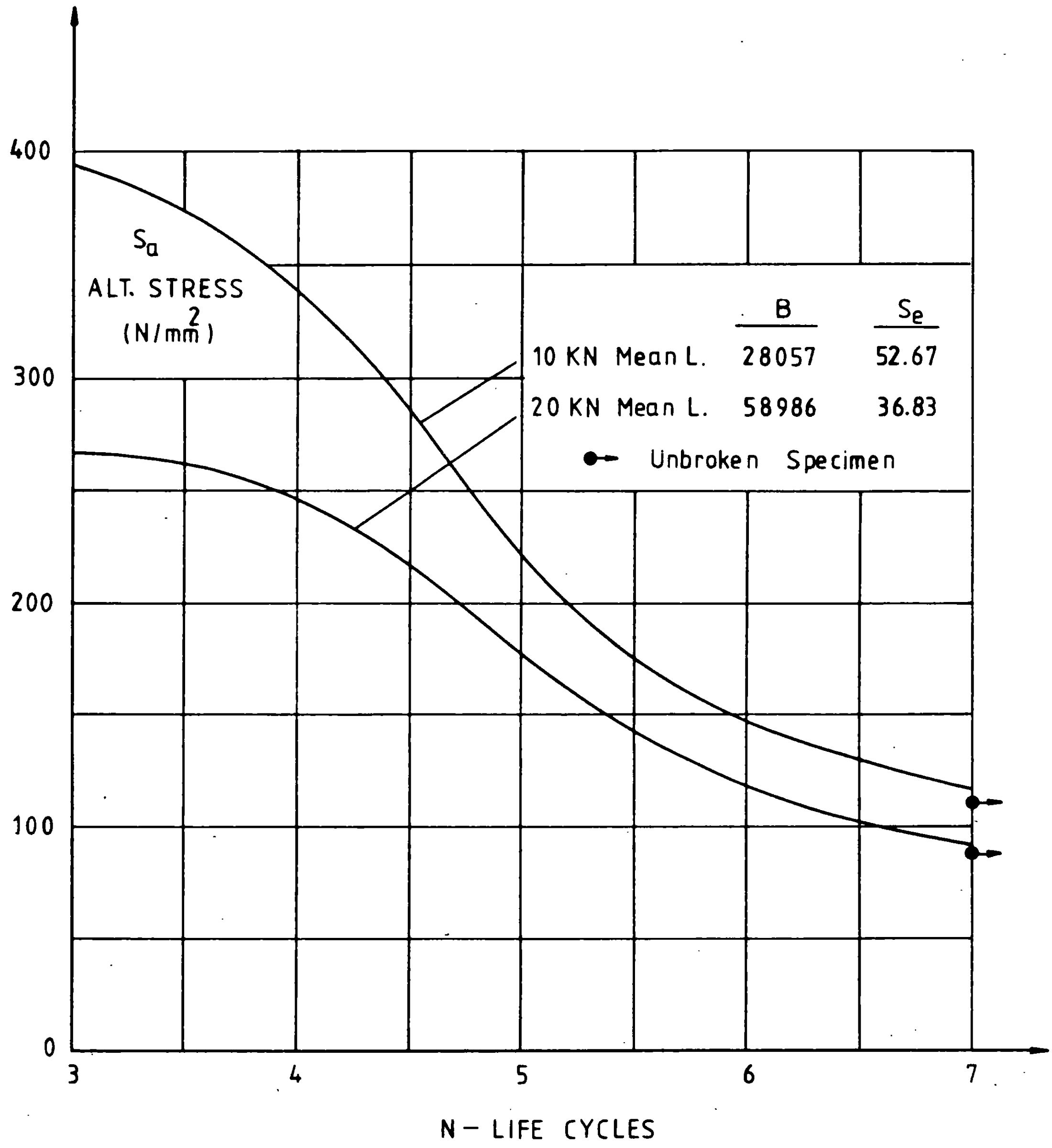


FIG.10.3 THEORETICAL MEAN LIFE OF THE SCREWED BAR (VIBRAPHORE)

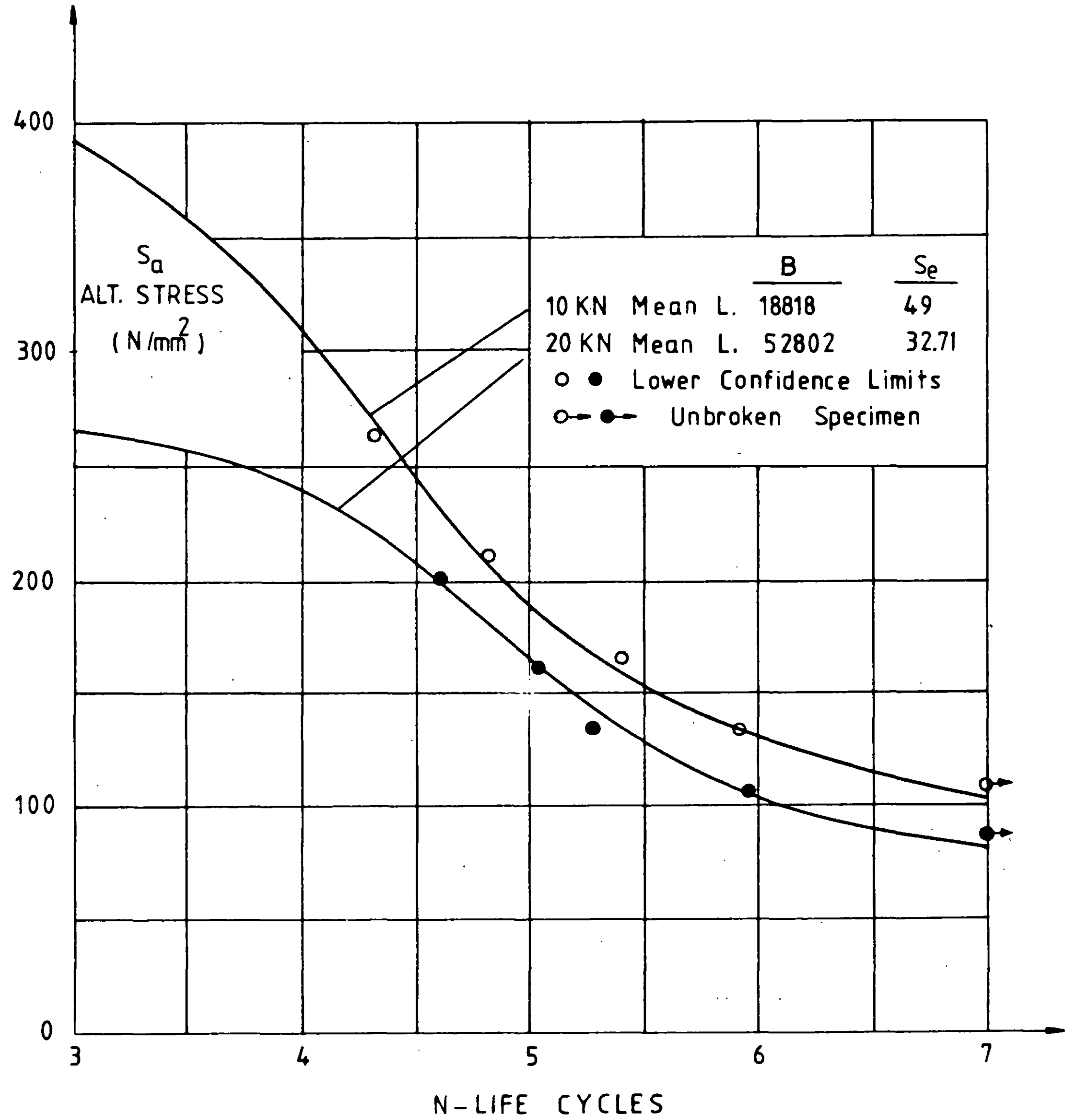


FIG.10_4 THEORETICAL LIFE OF THE SCREWED BAR AT 95% CONFIDENCE LEVEL (VIBRAPHORE)

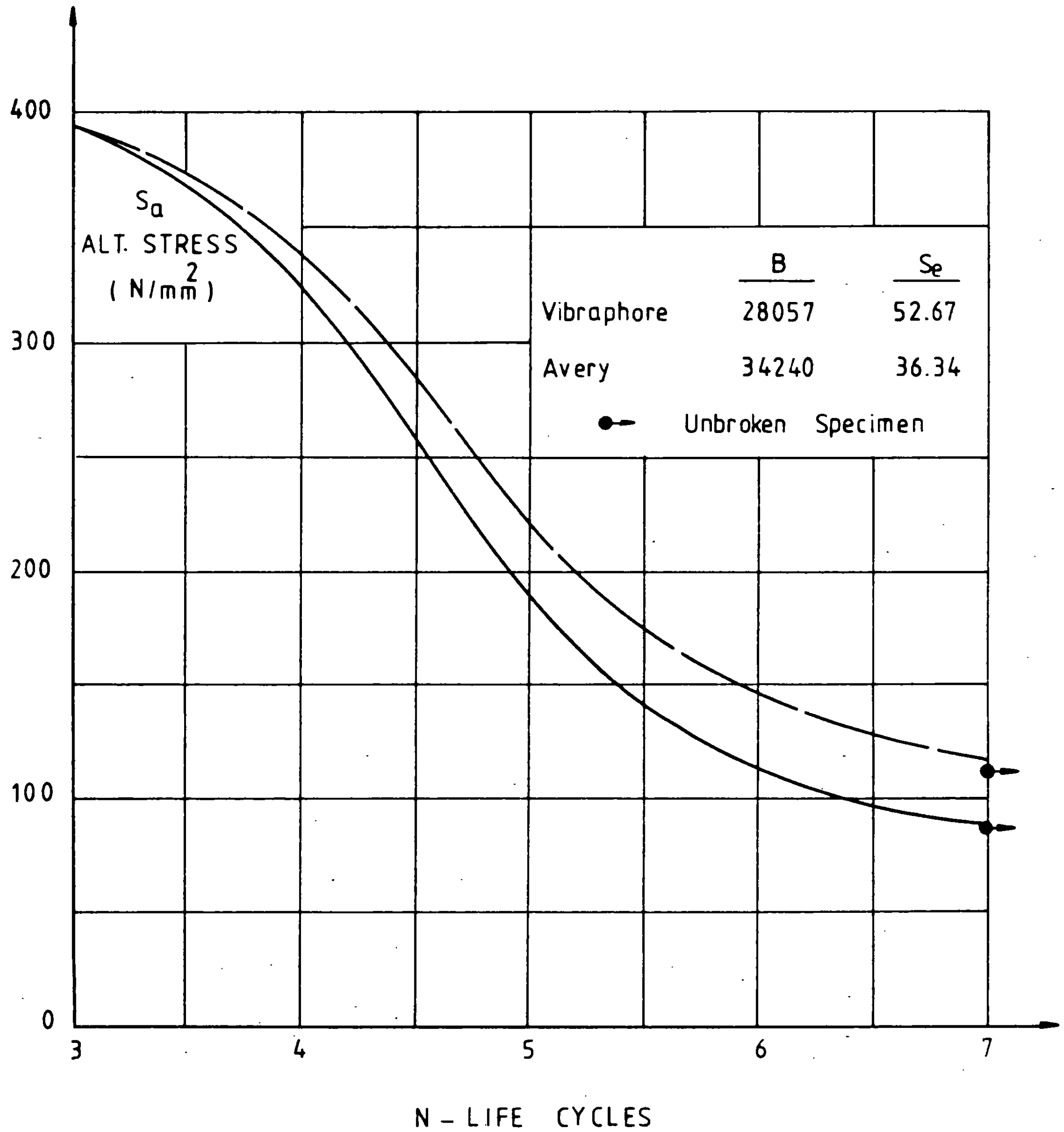


FIG.10.5 COMPARISON OF MEAN LIFE OF THE SCREWED BAR BETWEEN THE TWO MACHINES (10 KN-MEAN LOAD)

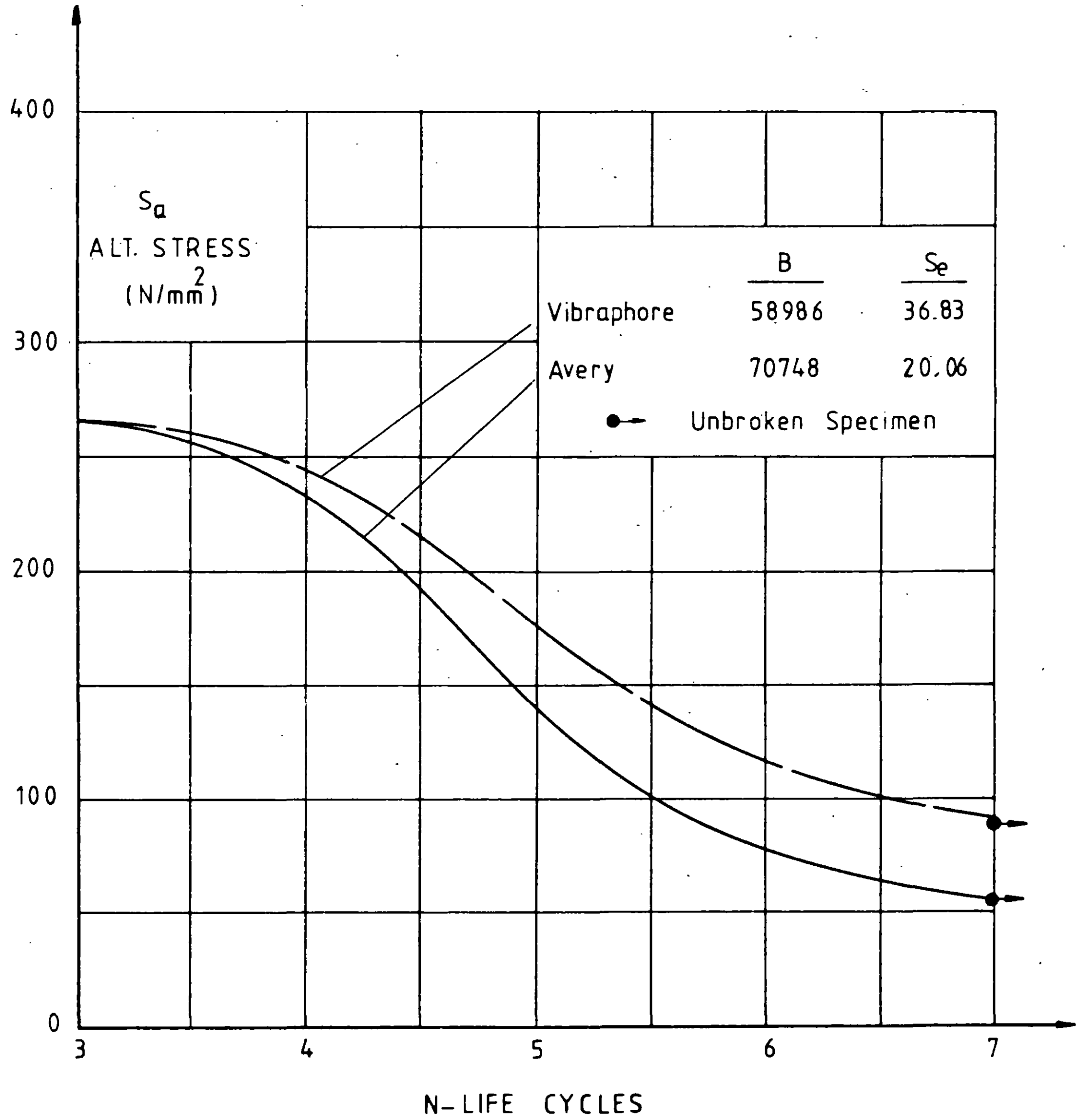


FIG.10_6 COMPARISON OF MEAN LIFE OF THE SCREWED BAR BETWEEN THE TWO MACHINES (20 KN-MEAN LOAD)

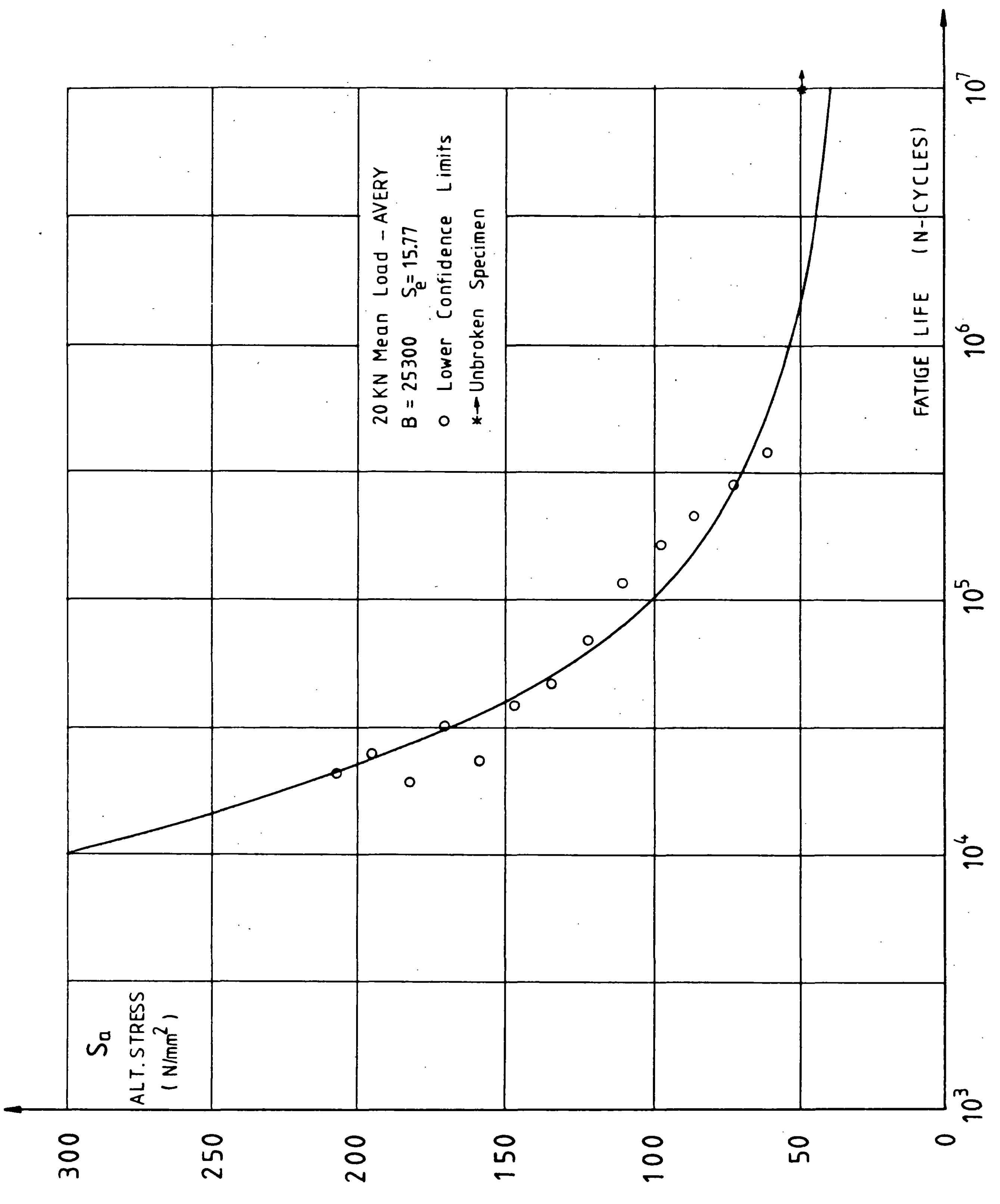


FIG.10_7 THEORETICAL LIFE OF THE Ø12 BOLT AT 95% CONFIDENCE LEVEL

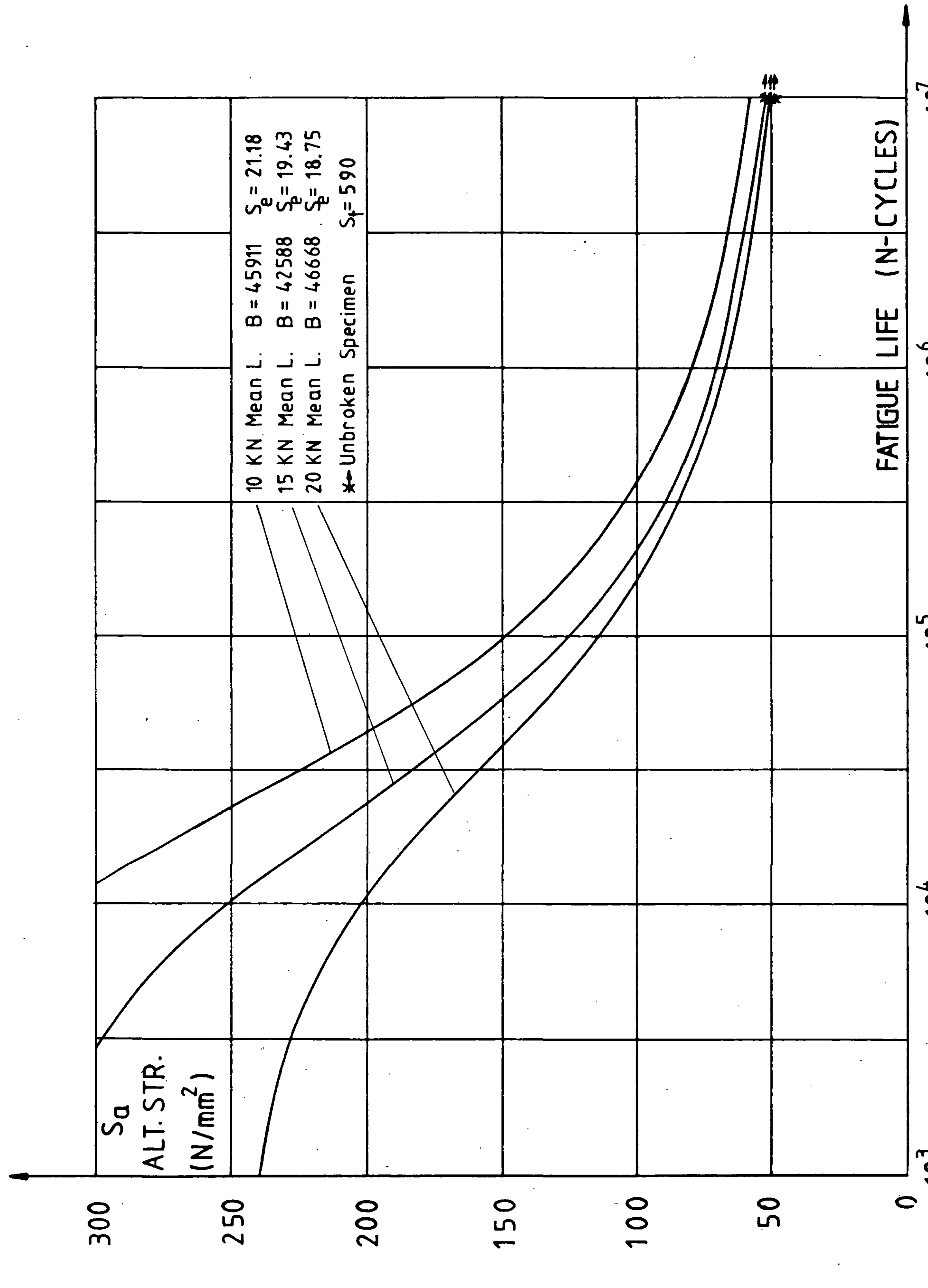


FIG.10_8 THEORETICAL MEAN LIFE OF THE
Ø10 BOLTS (VIBRAPHORE)

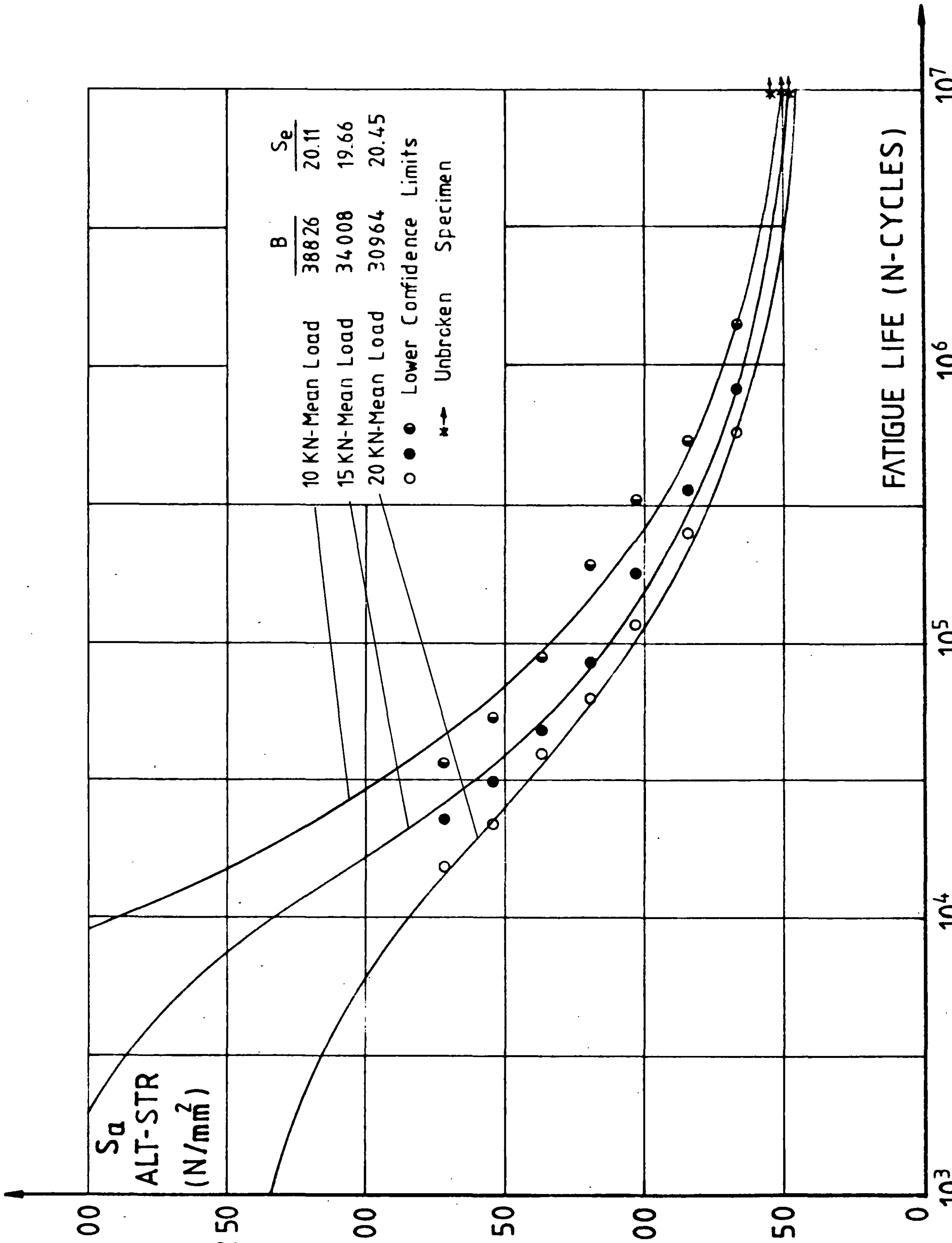


FIG.10_9 THEORETICAL LIFE OF THE Ø10 BOLT AT 95% CONFIDENCE LEVEL (VIBRAPHORE)

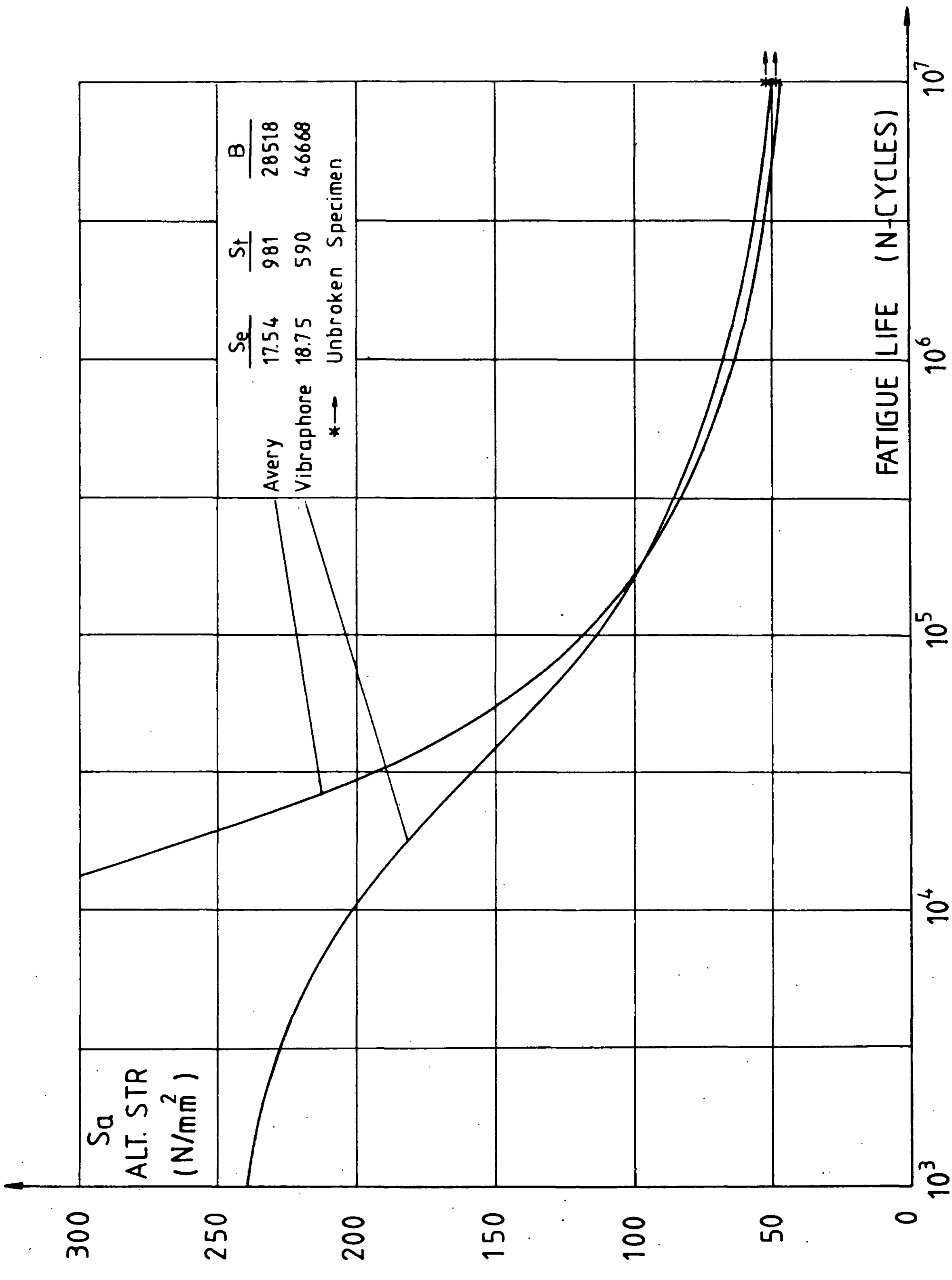


FIG.10_10 COMPARISON OF THE MEAN LIFE
OF THE Ø12 AND OF Ø10 BOLTS
IN THE TWO MACHINES
(20 KN-MEAN LOAD)

APPENDICES

APPENDIX A

PROGRAM FOR EVALUATION OF CONSTANTS B AND S_e IN JEFFERSON'S EQUATION

```
80 REM EVALUATION OF FATIGUE CONSTANT B
82 DIM B(20):DIM S(20)
85 A$="TENSILE STRENGTH      ="
90 B$="MEAN STRESS           ="
95 C$="ALTERNATING STRESS 1="
100 D$="FAILURE CYCLES 1     ="
105 E$="ALTERNATING STRESS 2="
110 F$="FAILURE CYCLES 2     ="
115 G$="EQUATION CONSTANT B ="
120 H$="ENDURANCE STRESS     ="
125 PRINT
130 PRINT"EVALUATION OF FATIGUE EQN CONSTANT B"
135 PRINT A$;:INPUT TS
140 PRINT B$;:INPUT SM
145 PRINT C$;:INPUT A1
150 PRINT D$;:INPUT N1
155 PRINT E$;:INPUT A2
160 PRINT F$;:INPUT N2
165 B(1)=10000
170 PRINT"ITERATIONS FOR B"
175 B(2)=B(1)+2000
180 FOR I=1 TO 20
185 PRINT B(I)
190 IF B(I)<=0 THEN B(I)=100
195 X1=LOG(B(I))
200 X2=LOG(N1+B(I))
205 X3=LOG(N2+B(I))
210 X4=(A1-A2)*(TS-SM)*X2*X3
215 X5=A2*(TS-SM-A1)*X2*X1
220 X6=A1*(TS-SM-A2)*X3*X1
225 S(I)=X4+X5-X6
230 IF I>1.0 THEN 250
235 IF S(I)>0 THEN B(2)=B(1)+2000
240 IF S(I)<0 THEN B(2)=B(1)-3000
245 GOTO 265
250 B(I+1)=B(I-1)-S(I-1)*(B(I-1)-B(I))/(S(I-1)-S(I))
255 BC=INT(B(I+1))
260 IF ABS(S(I))<0.1 THEN 270
265 NEXT
270 X7=A1*(TS-SM)*(LOG(N1+BC)-LOG(BC))
275 X8=(TS-SM)*LOG(N1+BC)-A1*LOG(BC)
280 SE=INT(100*X7/X8)/100
281 OPEN4,4
282 PRINT#4,CHR$(1)"EVALUATION OF FATIGUE CONSTANT B"
283 PRINT#4:PRINT#4:PRINT#4
285 PRINT#4,G$;:PRINT#4,BC
290 PRINT#4,H$;:PRINT#4,SE
295 PRINT#4,A$;:PRINT#4,TS
300 PRINT#4,B$;:PRINT#4,SM
305 PRINT#4
306 CLOSE4
310 X9=1-SE/(TS-SM)
320 N(1)=1E3
330 OPEN 4,4,0
340 CMD4
350 FOR J=1 TO 8
360 AS(J)=SE/(1-X9*LOG(BC)/LOG(N(J)+BC))
370 PRINT#4,N(J),:PRINT#4,AS(J)
380 N(J+1)=N(J)*10
390 NEXT J
400 PRINT#4
410 CLOSE4
READY.
```

APPENDIX B

PROGRAM FOR PLOTTING THE BEST FITTING JEFERSON'S CURVE

```
c         ****************************************************
c         THIS PROGRAM IS PLOTTING GRAPHICS
c         ****************************************************
c
          real nak,n1,n2,n3
          dimension sak(220),nak(220),st1(10),st2(10),cnt1(10),cnt2(10),cnt3(10),
$         ct1(10),ct2(10)
          external plot_$scale(descriptors),plot_$setup(descriptors),plot_(descriptors)
          k=0
          i=0
          n=0
          read(35,503)se,st,sm,b
   503    format(v)
     6    if(n-3.0e4)20,3,3
    20    n=n+1000
          go to 8
     3    if(n-4.0e5)30,9,9
    30    n=n+4000
          go to 8
     9    n=n+100000
     8    sa=se/(1.0-(1.0-se/(st-sm))*alog10(b)/alog10(b+n))
c         ****************************************************
c
          i=i+1
          sak(i)=sa
          nak(i)=alog10(n)
          if(n-1.0e7)6,7,7
     7    do 14 kn=1,10
          read(35,503)s1,s2,n1,n2,n3,c1,c2
          k=k+1
          st1(k)=s1
          st2(k)=alog10(s2)
          cnt1(k)=alog10(n1)
          cnt2(k)=alog10(n2)
          cnt3(k)=alog10(n3)
          ct1(k)=alog10(c1)
          ct2(k)=alog10(c2)
    14    continue
c
c
c         ****************************************************
c
          call plot_$setup('JEFFERSON-S EMP.S-N CURVE','N_LIFE CYCLES','
$         Sa ALT.ST N/mm2',1,base,2,0)
          call plot_$scale(3.0,7.0,0.0,400.0)
          call plot_(nak,sak,218,1,'.')
          call plot_(st2,st1,10,3,'*')
          call plot_$setup('JEFFERSON-S EMP.S-N CURVE','N_LIFE CYCLES','
$         Sa ALT.ST N/mm2',1,base,2,0)
          call plot_$scale(3.0,7.0,0.0,400.0)
          call plot_(nak,sak,218,1,'.')
          call plot_(cnt1,st1,10,3,'*')
          call plot_(cnt2,st1,10,3,'*')
          call plot_(cnt3,st1,10,3,'*')
          call plot_(ct1,st1,10,3,'.')
          call plot_(ct2,st1,10,3,'.')
c         ****************************************************
          stop
          end
```

APPENDIX C

Sample Calculations of Log-normal Analysis for $\phi 10$ Bolts

The table below is prepared by the data taken from table 6.2 at 10 KN mean load and 4 KN alternating load level :

Test No	Cycle Life x	Log Cycle Life x_l	x_l^2	Median Rank $y=f(x)$	y^2	$x_l \cdot y$
1	5.300×10^5	5.724	32.764	0.0943	0.00889	0.539
2	6.492×10^5	5.812	33.769	0.2295	0.05267	1.334
3	7.430×10^5	5.871	34.469	0.3648	0.01330	2.147
4	7.978×10^5	5.902	34.834	0.5000	0.25000	2.951
5	8.496×10^5	5.929	35.153	0.6352	0.40348	3.766
6	10.293×10^5	6.013	36.156	0.7705	0.59367	4.633
7	10.538×10^5	6.023	36.277	0.9057	0.82029	5.455
Sum $\Sigma=$		41.274	243.432	3.5	2.26208	20.825

In order to find the best fitting straight line of the form :

$$y = a_0 + a_1 x$$

the coefficients, a_0 and a_1 are evaluated by means of the following formulae

$$a_0 = \frac{\Sigma y \, \Sigma x_l^2 - \Sigma x_l \, \Sigma x_l \cdot y}{n \Sigma x_l^2 - (\Sigma x_l)^2} = \frac{(3.5)(243.432)^2 - (41.274)(20.825)}{7(243.432) - (41.274)^2} = -15.2$$

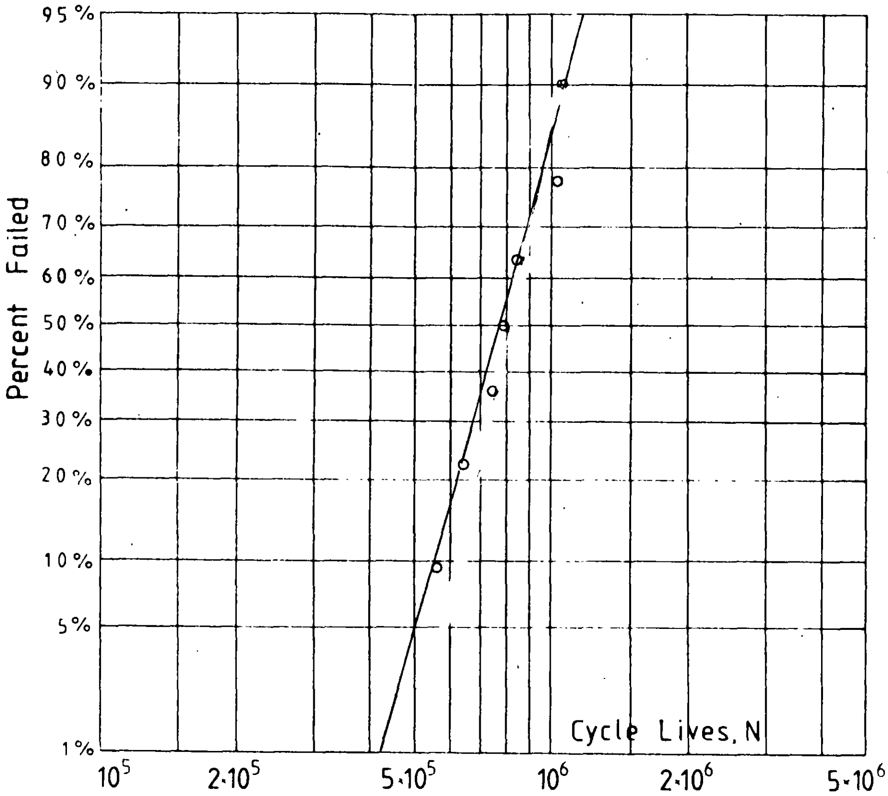
$$a_1 = \frac{n \Sigma x_l \cdot y - \Sigma x_l \, \Sigma y}{n \Sigma x_l^2 - (\Sigma x_l)^2} = \frac{7(20.825) - (41.274)(3,5)}{7(243.432) - (41.274)^2} = 2.66$$

$\therefore$ The equation of the best fitting straight line is found to be :

$$y = -15.2 + 2.66 x_l$$

In the following figure log-normal plot of the cycle lives is shown.

together with the best fitting straight line to the data.



LOG-NORMAL PLOT OF Ø10-BOLT LIVES

The correlation coefficient of the best fitting line is computed by the following formula:

$$r = \frac{n\sum x_i \cdot y - \sum x_i \sum y}{\left\{\left[n\sum x_i^2 - (\sum x_i)^2\right]\left[n\sum y^2 - (\sum y)^2\right]\right\}^{1/2}}$$

$$r = \frac{7(20.825) - (41.274)(3.5)}{\left\{\left[7(243.432)^2 - (41.274)^2\right]\left[7(2.26208 - (3.5)^2\right]\right\}^{1/2}} = 0.978$$

$$r = 97.8\ \%$$

Sample Calculations of Weibull Analysis for ϕ10 Bolts

Using the same data as in the log-normal analysis the table below is prepared.

Test No	Cycle Life x	Ln Cycle Life $X = \ln x$	X^2	Median Rank $y = f(x)$	$Y = \ln\ln \frac{1}{1-f(x)}$	XY
1	5.300×10^5	13.181	173.739	0.0943	-2.312	-30.474
2	6.432×10^5	13.383	179.105	0.2295	-1.344	-17.987
3	7.430×10^5	13.518	182.736	0.3648	-0.790	-10.679
4	7.978×10^5	13.590	184.688	0.5000	-0.367	-4.988
5	8.496×10^5	13.653	186.404	0.6352	0.008	0.114
6	10.293×10^5	13.844	191.656	0.7705	0.387	5.358
7	10.538×10^5	13.868	192.321	0.9057	0.859	11.915
Sum $\Sigma =$		95.037	1290.649	3.5	-3.558	-46.741

In order to find the best fitting straight line of the form:

$$Y = a_0 + a_1 X$$

the coefficients a_0 and a_1 are evaluated by means of the following formulae:

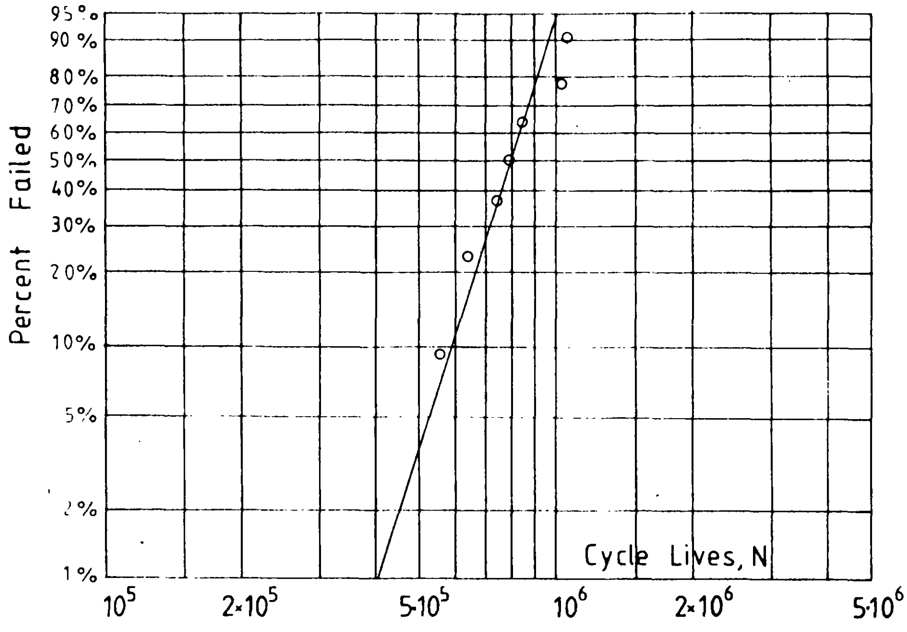
$$a_0 = \frac{\Sigma Y \, \Sigma X^2 - \Sigma X \, \Sigma XY}{n \Sigma X^2 - (\Sigma X)^2} = \frac{(1290.649)(-3.558) - (95.037)(-46.741)}{7(1290.649) - (95.037)^2} = 59.73$$

$$a_1 = \frac{n \Sigma XY - \Sigma X \Sigma Y}{n \Sigma X^2 - (\Sigma X)^2} = \frac{7(-46.741) - (95.037)(-3.558)}{7(1290.649) - (95.037)^2} = 4.36$$

$\therefore$ The equation of the best fitting straight line is found to be:

$$Y = 59.73 + 4.36\, X$$

In the following figure Weibull plot of the cycle lives is shown together *with the best fitting straight line to the data.*



WEIBULL PLOT OF Ø10-BOLT LIVES

The correlation coefficient of the best fitting line is computed by the following formula :

$$r = \frac{n\Sigma XY - \Sigma X \Sigma Y}{\{[n\Sigma X^2 - (\Sigma X)^2][n\Sigma Y^2 - (\Sigma Y)^2]\}^{1/2}}$$

$$r = \frac{7(-46.741) - (95.037)(-3.558)}{\{[7(1290.649) - (95.037)^2][7.(8.798) - (-3.558)^2]\}^{1/2}} = 0.988$$

$$r = 98.8\ \%$$

APPENDIX D

DERIVATION OF EQUATION 9.13

Starting from the original Jefferson's Equation :

$$S_a = \frac{S_e}{1-\left(1-\frac{S_e}{S_t-S_m}\right)\cdot\frac{\log B}{\log(N+B)}} \qquad D.1$$

Rearranging it for Se

$$S_e = S_a - S_a\,\frac{\log B}{\log(N+B)} + \frac{S_a\cdot S_e}{S_t-S_m}\cdot\frac{\log B}{\log(N+B)} \qquad D.2$$

$$S_e\left[1-\frac{S_a}{S_t-S_m}\cdot\frac{\log B}{\log(N+B)}\right] = S_a\left[1-\frac{\log B}{\log(N+B)}\right] \qquad D.3$$

$$S_e = \frac{S_a\left[1-\frac{\log B}{\log(N+B)}\right]}{1-\frac{S_a}{S_t-S_m}\cdot\frac{\log B}{\log(N+B)}} \qquad D.4$$

$$S_e = \frac{S_a\left[\log(N+B)-\log B\right]/\left[\log(N+B)\right]}{(S_t-S_m)\left[\log(N+B)-S_a\log B\right]/(S_t-S_m)\cdot\log(N+B)} \qquad D.5$$

Cancelling log(N+B) in equation D.5 the following is obtained:

$$S_e = \frac{S_a\,(S_t-S_m)\left[\log(N+B)-\log B\right]}{(S_t-S_m)\cdot\log(N+B)-S_a\cdot\log B} \qquad D.6$$

Replacing, as the boundary conditions, the coordinates of the two points , $P_1(S_{a1},N_1)$, $P_2(S_{a2},N_2)$ on the same curve into D.6 :

For $P_1(S_{a1},N_1) \longrightarrow$ $$S_e = \frac{S_{a1}(S_t-S_m)\left[\log(N_1+B)-\log B\right]}{(S_t-S_m)\cdot\log(N_1+B)-S_{a1}\log B} \qquad D.7$$

For $P_2(S_{a2},N_2) \longrightarrow$ $$S_e = \frac{S_{a2}(S_t-S_m)\left[\log(N_2+B)-\log B\right]}{(S_t-S_m)\cdot\log(N_2+B)-S_{a2}\log B} \qquad D.8$$

Dividing D7 by D8

$$1=\frac{\left[S_{a1}(S_t-S_m)\log(N_1+B)-S_{a1}(S_t-S_m)\log B\right]\left[(S_t-S_m)\log(N_2+B)-S_{a2}\log B\right]}{\left[(S_t-S_m)\log(N_1+B)-S_{a1}\log B\right]\left[S_{a2}(S_t-S_m)\log(N_2+B)-S_{a2}(S_t-S_m)\log B\right]} \qquad D9$$

Rearranging D.9

$$S_{a1}(S_t-S_m)^2\log(N_1+B)\log(N_2+B)-S_{a1}S_{a2}(S_t-S_m)\log B\log(N_1+B)-$$

$$S_{a1}(S_t-S_m)^2\log(N_2+B)\log B+S_{a1}S_{a2}(S_t-S_m)\log^2 B=S_{a1}S_{a2}(S_t-S_m)\log^2 B+$$

$$S_{a2}(S_t-S_m)^2\log(N_1+B)\log(N_2+B)-S_{a2}(S_t-S_m)\log(N_2+B)\log B-$$

$$S_{a1}S_{a2}(S_t-S_m)\log(N_2+B)\log B \qquad D.10$$

After cancelling and rearranging :

$$(S_{a1}-S_{a2})(S_t-S_m)\log(N_1+B)\log(N_2+B)+S_{a2}(S_t-S_m-S_{a1})\log(N_1+B)\log B$$

$$-S_{a1}(S_t-S_m-S_{a2})\log(N_1+B)=0 \qquad (9.13)$$

is finally obtained.